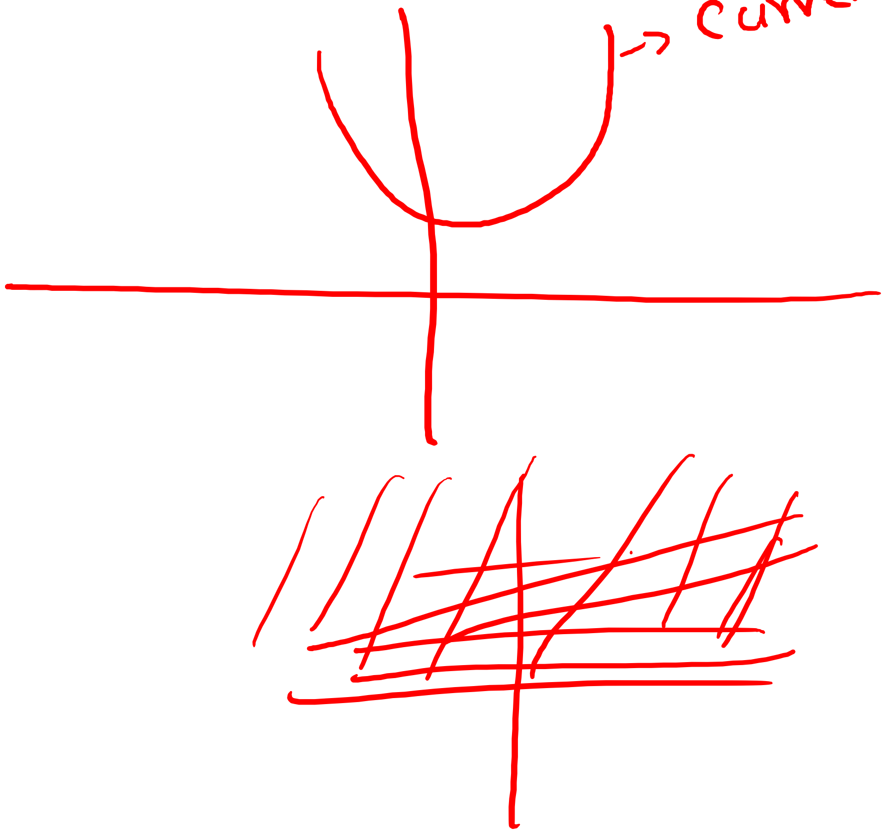
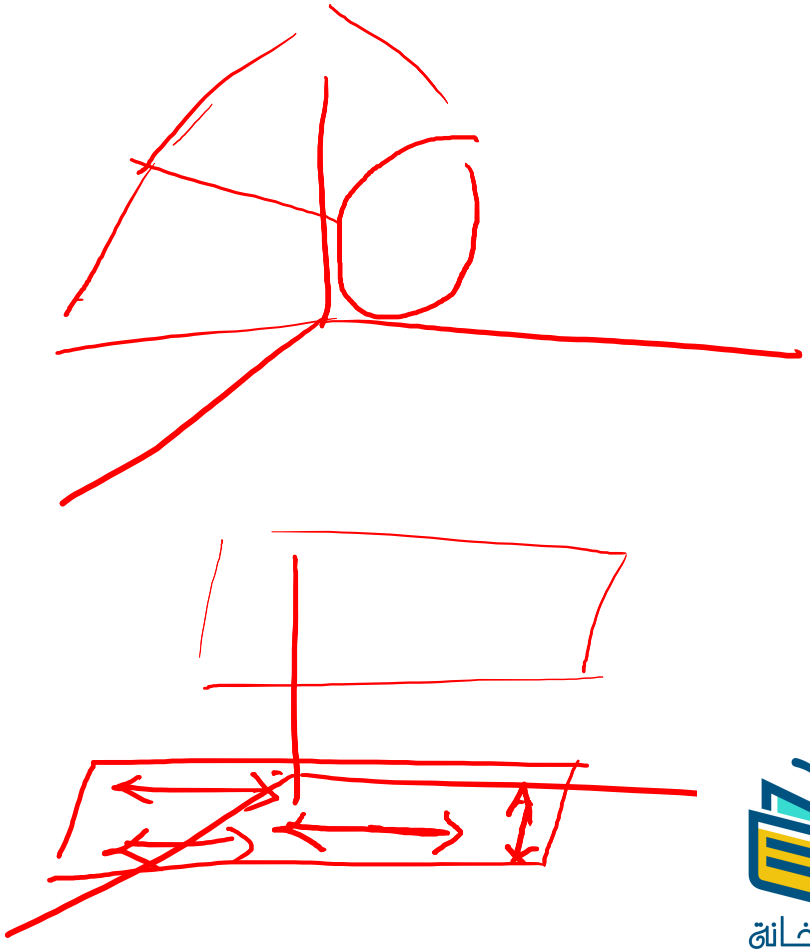
Lecture 1 Calculus 2/ Bilgi University (Spring 2022)

كورس خانة

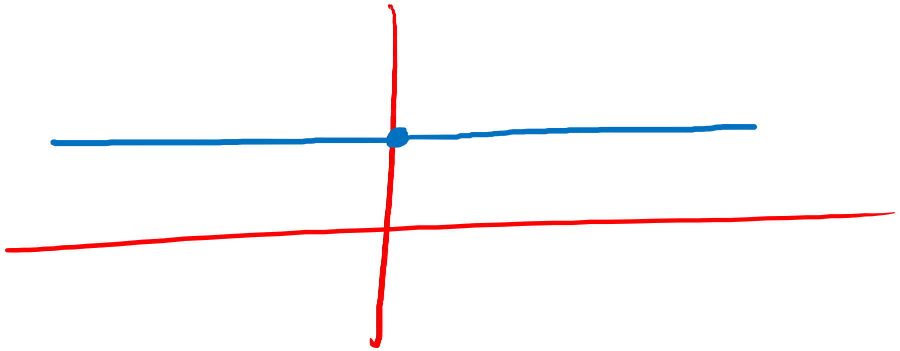
Course Khana

2D

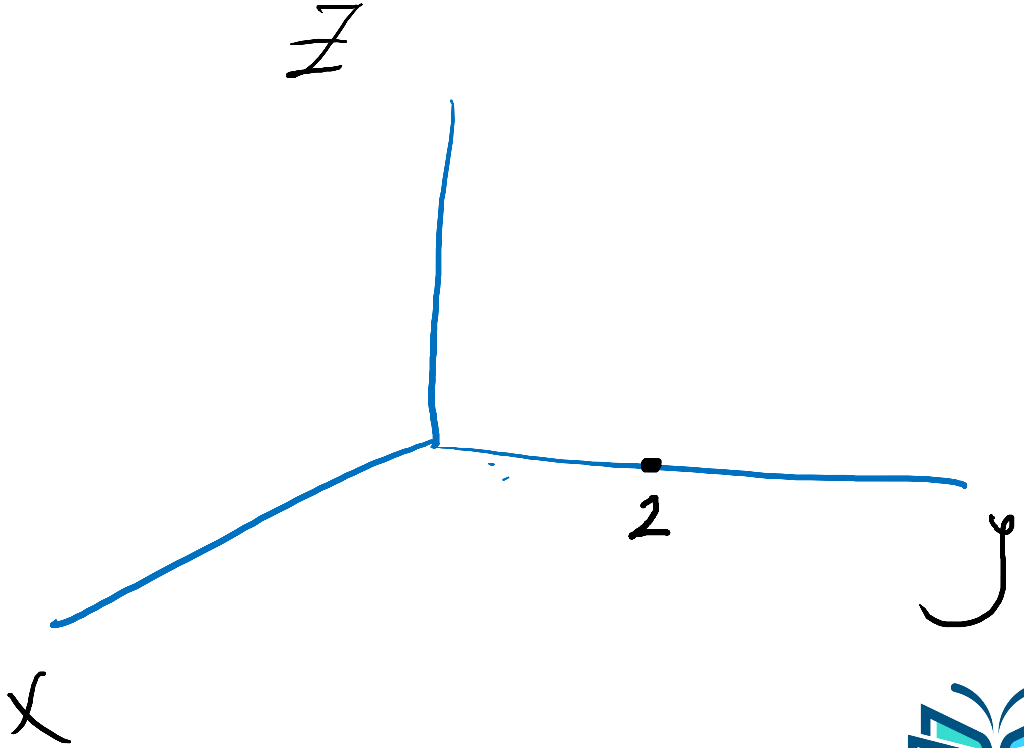




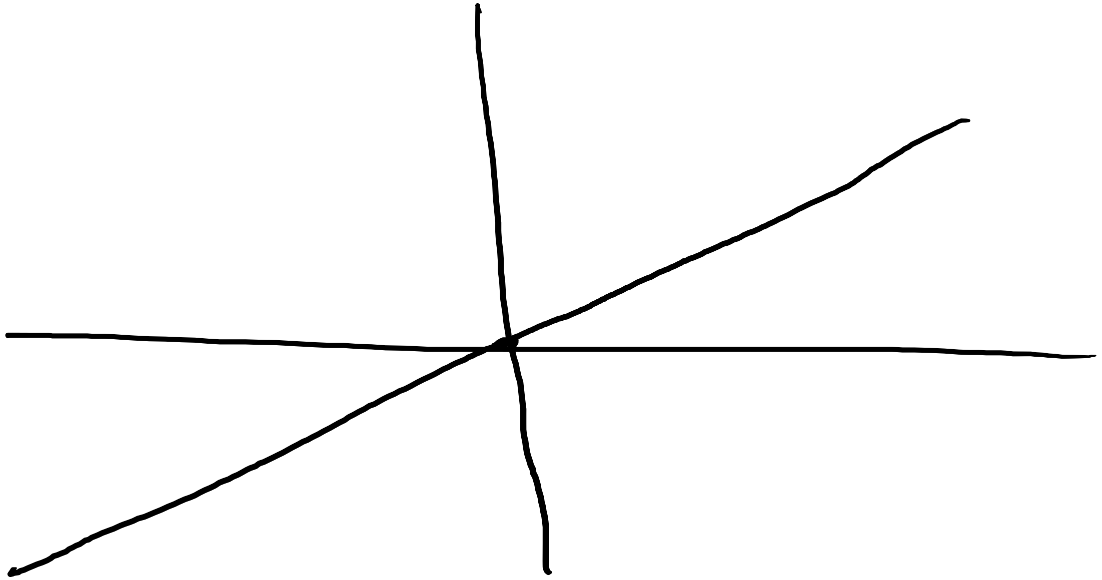
2D $y=2$



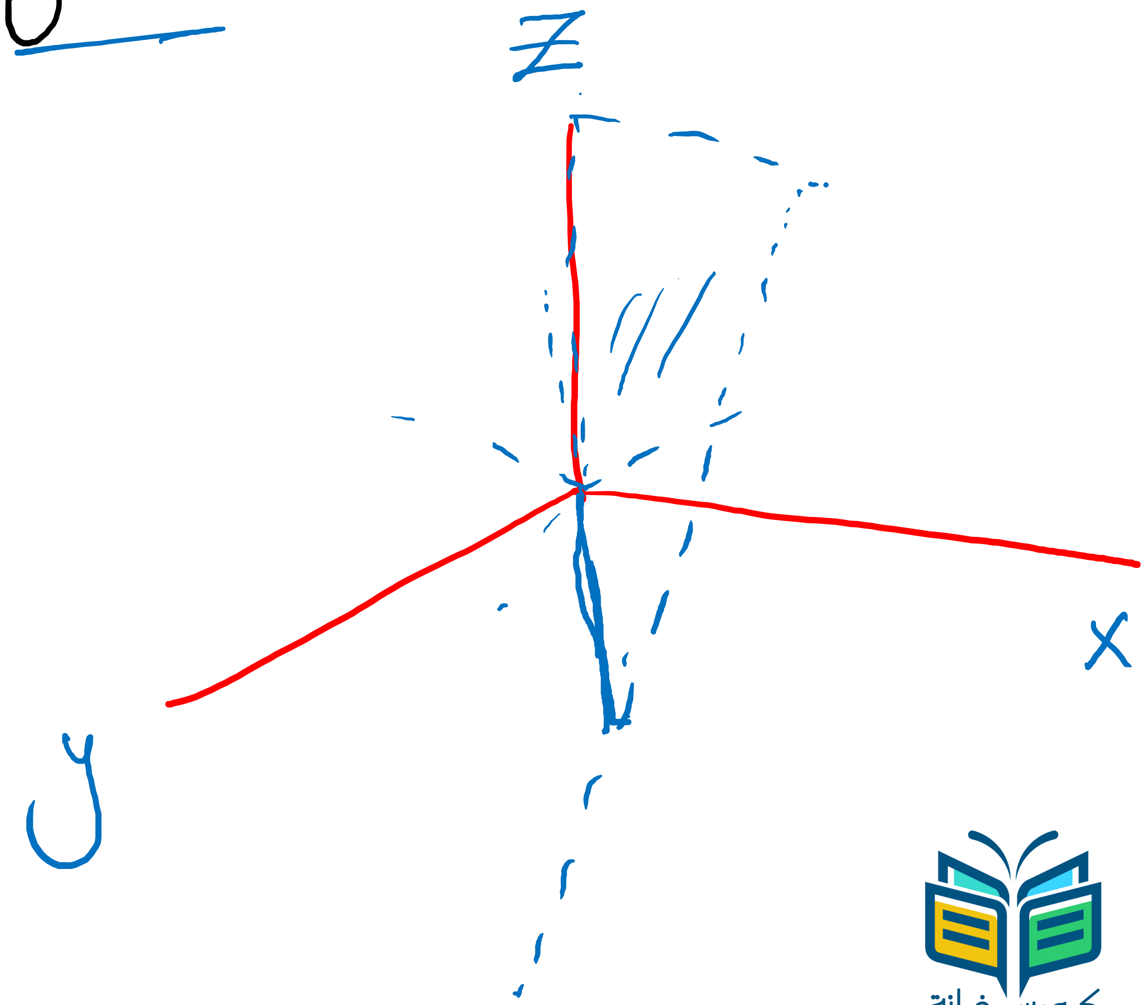
3D $y=2$



$y = x$



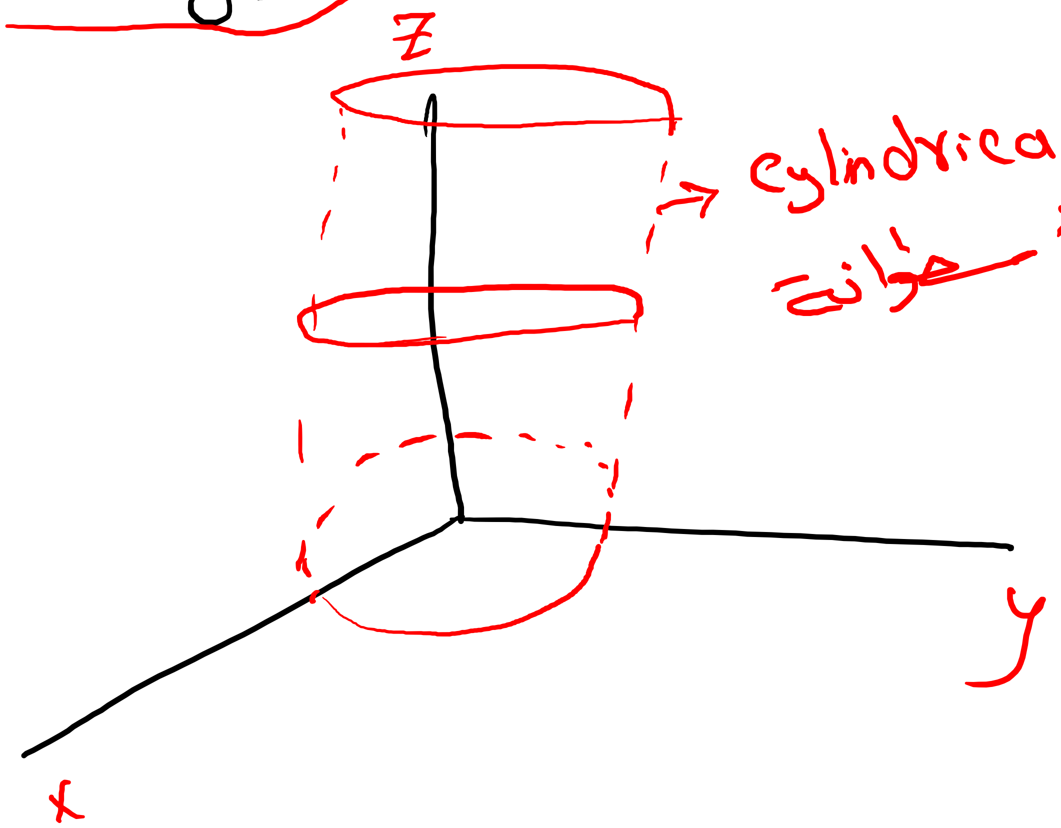
$y = x$ missing $\boxed{Z}$



3D

$x^2+y^2=25 \rightarrow$ in 2D circle

missing Z



cylindrical

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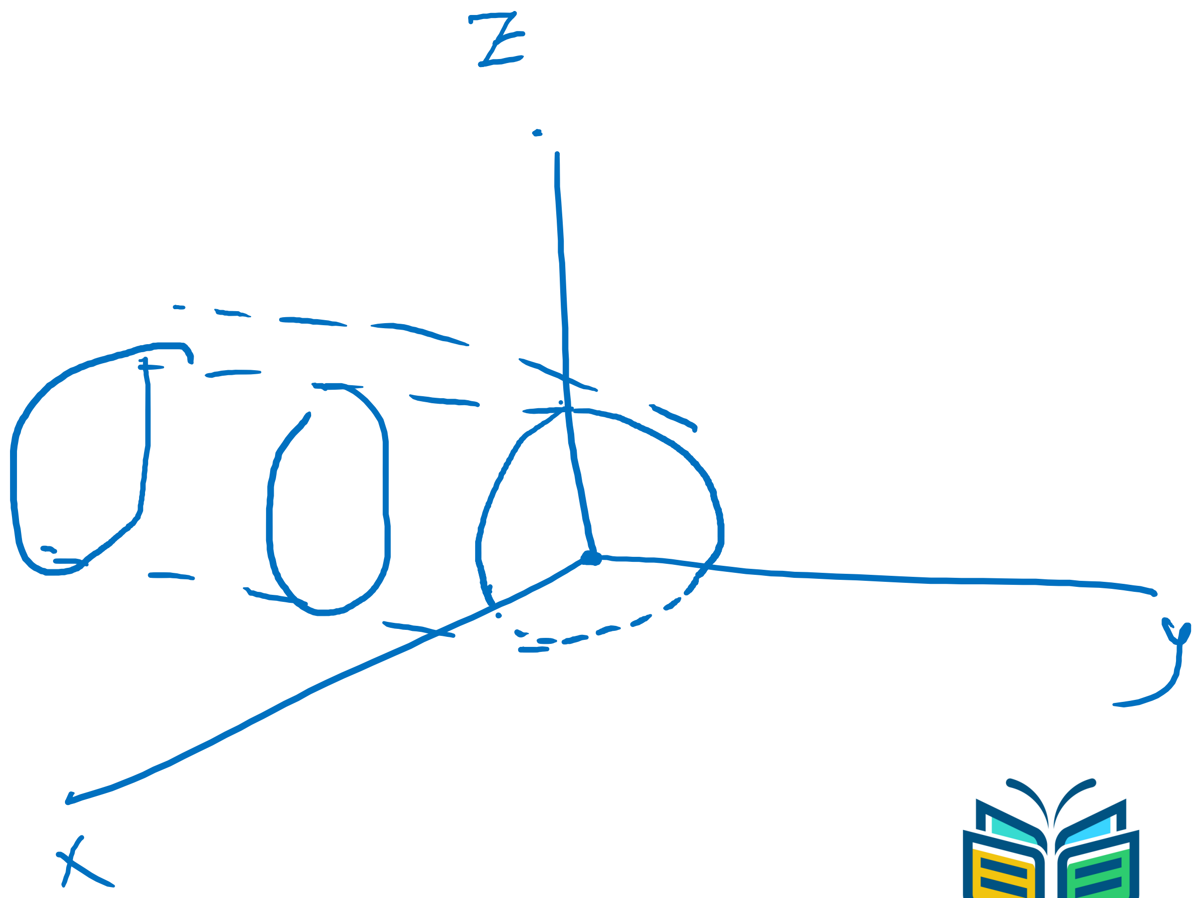
$x^2+Z^2=16$ 2D → circle



* Remember in 2D

equation of circle

$$(x-a)^2+(y-b)^2=r^2$$

$(a,b) \Rightarrow$ center of circle

$r \Rightarrow$ radius

in 3D

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equation of sphere:

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

$(a,b,c) \Rightarrow$ center of sphere

$r \Rightarrow$ radius of spher

Show that

$$x^2+y^2+z^2+4x-6y+2z+6=0$$

is an equation of sphere and find its center and radius

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

Complete square

$+1 \ -1 \ +5$

$$x^2+2x+5=(x^2+2x+1)+4$$

$$\frac{2}{2}=(1)^2=\boxed{1}$$

$$=(x+1)^2+4$$

$$x^2-4x=(x^2-4x+4)-4$$

$$\frac{-4}{2}=(-2)^2=4$$

$$=(x-2)^2-4$$

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$$x^2+y^2+z^2+4x-6y+2z+6=0$$

$$(x^2+4x+4)-4+(y^2-6y+9)-9+(z^2+2z+1)-1=-6$$

$\frac{4}{2}=(2)^2=4$ $\qquad$ $\frac{-6}{2}=(-3)^2=9$

-2

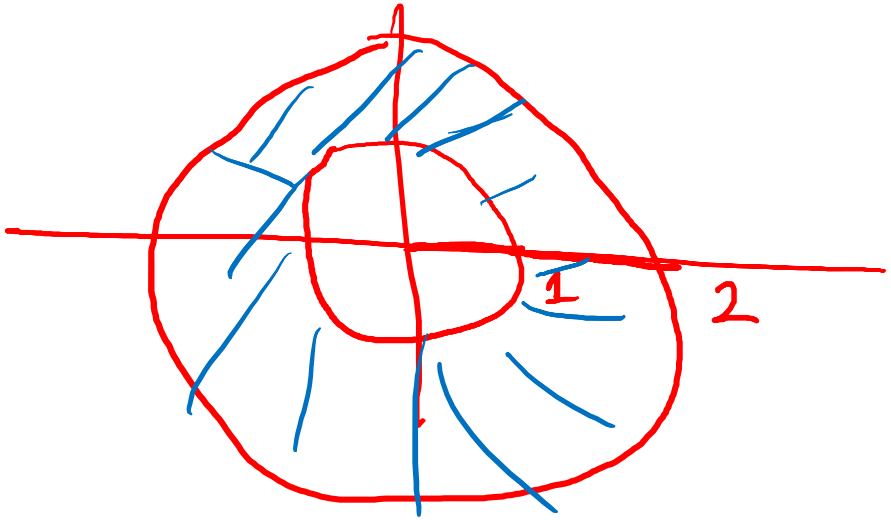
$$(x+2)^2+(y-3)^2+(z+1)^2=-6+4+9+1$$

$$(x+2)^2+(y-3)^2+(z+1)^2=8 \to r^2$$

center $(-2,3,-1)$ $\qquad$ $r=\sqrt{8}$

in 2D

$$1 \leq x^2+y^2 \leq 4$$



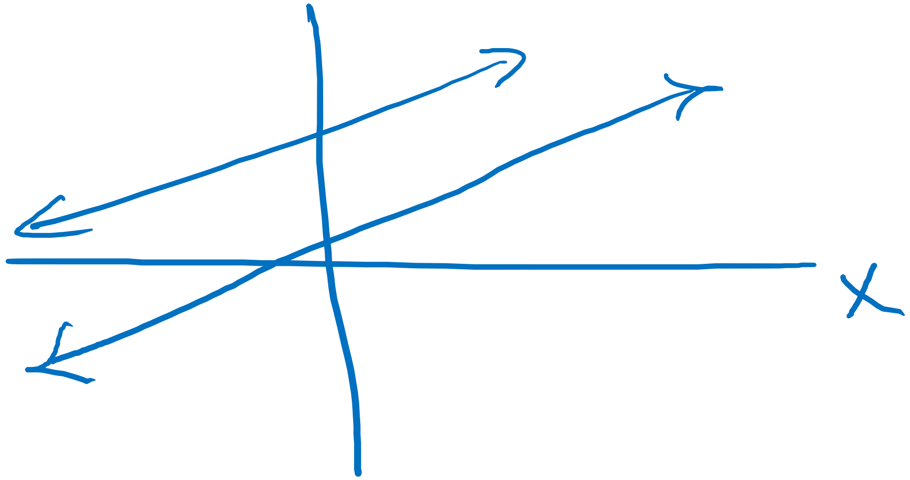
$$x^2+y^2=1$$

$$x^2+y^2=4$$

2D

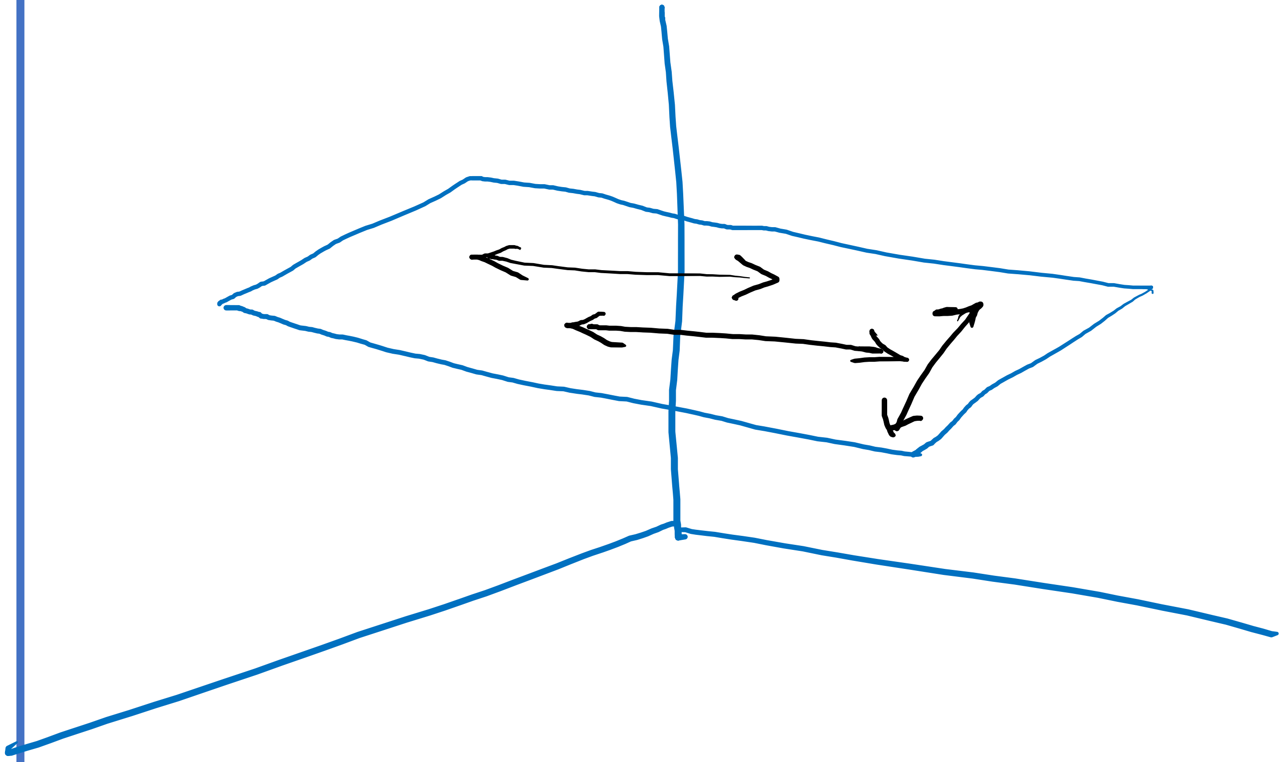
y

plane



3D

Space



2D Equation of lines

$$y = m(x - x_1) + y_1$$

$m \Rightarrow$ slope $\rightarrow$ direction

(x_1, y_1) Point on the line.

3D Equation of lines

1. Point $P_0(x_0, y_0, z_0)$
2. Direction (vector Parallel)

$$V = \langle a, b, c \rangle$$

1. Vector equation of line

$$(x, y, z) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$

$$\vec{r} = \vec{r_0} + t\vec{v}$$

2 Parametric equation of a line.

$$x = x_0 + at \Rightarrow t = \frac{x - x_0}{a}$$

$$y = y_0 + bt \Rightarrow t = \frac{y - y_0}{b}$$

$$z = z_0 + ct \Rightarrow t = \frac{z - z_0}{c}$$

3 Symmetric equation

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Hint

* $\vec{u} \perp \vec{v}$ if $\vec{u} \cdot \vec{v} = 0$

* $\vec{u} \parallel \vec{v}$ if $\vec{u} = c\vec{v}$

$$\langle 1, 2, 3 \rangle \parallel \langle 3, 6, 9 \rangle$$

$$\frac{3}{1} = \frac{6}{2} = \frac{9}{3} =$$

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ex

Is the line passes through

A (-4, -6, 1) and B (-2, 0, -3)

Parallel to the line through

C (10, 18, 4) and D (5, 3, 14) ??

Solution

$\overrightarrow{AB} = B - A = (-2, 0, -3) - (-4, -6, 1)$

$= \langle 2, 6, -4 \rangle$

$\overrightarrow{CD} = D - C = \langle -5, -15, 10 \rangle$

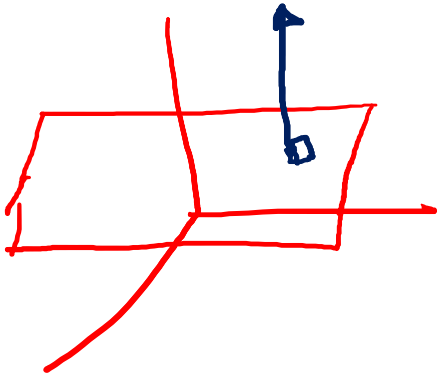
check

$\frac{-5}{2} \overset{?}{=} \frac{-\cancel{15}^{\,5}}{\cancel{6}_{\,2}} \overset{?}{=} \frac{10}{-4}$

$\frac{-5}{2} = \frac{-5}{2} = \frac{-5}{2}$

So they are Parallel to each other.

* Equation of a Plane



$\boxed{1}$ Point on the plane

$$P_0 = (x_0, y_0, z_0)$$

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$\boxed{2}$ Normal to the plane

$$\vec{n} = \langle a, b, c \rangle$$

Equation of a Plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Example 19

Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\vec{n} = \langle 2, 3, 4 \rangle$. Find the intercepts and sketch the plane.

$$P_0 = (\overset{x_0}{2}, \overset{y_0}{4}, \overset{z_0}{-1}) \qquad \vec{n} = \langle \overset{a}{2}, \overset{b}{3}, \overset{c}{4} \rangle$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$2(x-2) + 3(y-4) + 4(z+1) = 0$$

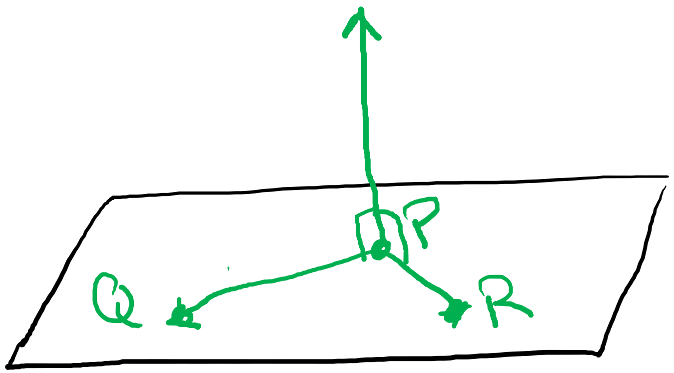
$$2x - \cancel{4} + 3y - 12 + 4z + \cancel{4} = 0$$

$$\boxed{2x + 3y + 4z = 12}$$

Example 20

Find an equation of the plane that passes through the points $P(1,3,2)$, $Q(3,-1,6)$ and $R(5,2,0)$.

$$\begin{array}{c} \quad\ x_0\ \ y_0\ \ z_0 \\ P_0 = (1,3,2) \end{array} \qquad \vec{n} = ??$$



$$\overrightarrow{PQ} \times \overrightarrow{PR} = \vec{n}$$

$$\overrightarrow{PQ} = Q - P = \langle 2, -4, 4 \rangle$$

$$\overrightarrow{PR} = R - P = \langle 4, -1, -2 \rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & k \\ 2 & -4 & 4 \\ 4 & -1 & -2 \end{vmatrix} \end{array}$$

$$= (8+4)i - \overset{-20}{(-4-16)}j + (-2+16)k$$

$$\vec{n} = 12i + 20j + 14k$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$12(x-1)+20(y-3)+14(z-2)=0$$

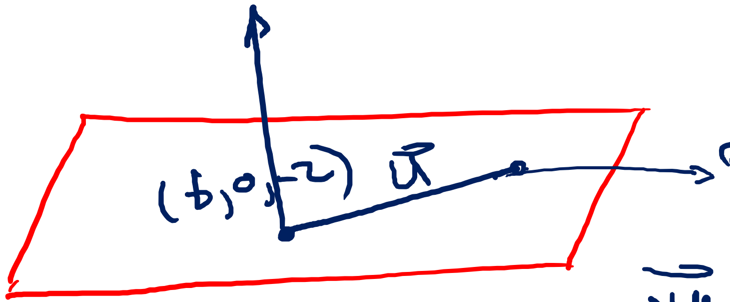
$$12x-12+20y-60+14z-28=0$$

$$12x+20y+14z=12+60+28$$

$$12x+20y+14z=100$$

(d) The plane that passes through the point $(6, 0, -2)$ and contains the line $x = 4-2t$, $y = 3+5t$, $z = 7 + 4t$.

$P_0 = (6, 0, -2) \qquad n = ??$



$P_1 = (4, 3, 7)$

$\vec{v} = \langle -2, 5, 4 \rangle$

$\vec{u} = (4, 3, 7) - (6, 0, -2)$

$= \langle -2, 3, 9 \rangle$

$\vec{u} \times \vec{v} = \vec{n}$ ✓✓

* Line of intersection between two plane.

direction

$\vec{n}_1 \times \vec{n}_2 =$

* Angle between two planes

is the angle between $\vec{n}_1, \vec{n}_2$

$\vec{n}_1 \cdot \vec{n}_2 = |n_1||n_2| \cos\theta$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|n_1|\,|n_2|}$$

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|n_1| \cdot |n_2|}\right)$$

. Let $x + y + z = 1$ and $x + 2y + 2z = 1$ be two planes. Find parametric equations for the line of intersection of the planes and find the angle between the planes.

Point, direction

$\vec{n}_1 = \langle 1,1,1 \rangle$, $\vec{n}_2 = \langle 1,2,2 \rangle$

$\vec{V} = \vec{n}_1 \times \vec{n}_2$

$$= \begin{array}{ccc} + & - & + \end{array}$$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$= (2-2)i - (2-1)j + (2-1)k$

$= 0i - j + k$

$a = 0$

$b = -1$

$c = 1$

let $Z = 0$

$x + y + z = 1$

$x + 2y + 2z = 1$

$x + y = 1$

$- \; x + 2y = 1$

$\Rightarrow \boxed{y = 0}$, $\boxed{x = 1}$

Point on the line of intersection

$\begin{matrix} x_0 & y_0 & z_0 \end{matrix}$
$(1, 0, 0)$

$x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

$x = 1 + 0t$, $y = 0 + (-1)t$, $z = 0 + t$

$x = 1$, $y = -t$, $z = t$

* for the angle

$$\vec{n}_1 \cdot \vec{n}_2 = |n_1|\,|n_2| \cos\theta$$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|n_1|\,|n_2|}$$

$$\vec{n}_1 \cdot \vec{n}_2 = \langle 1,1,1 \rangle \cdot \langle 1,2,2 \rangle = 1+2+2$$
$$= 5$$

$$|\vec{n}_1| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$$

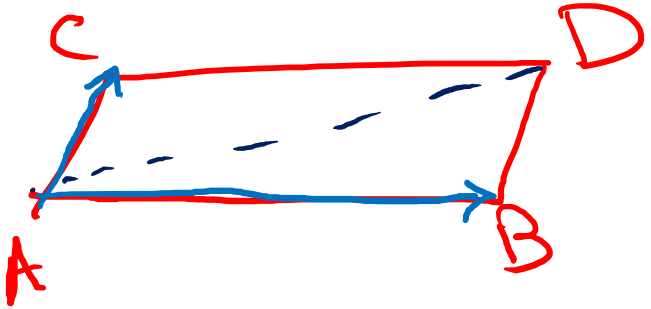
$$|\vec{n}_2| = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$$

$$\cos\theta = \frac{5}{9\sqrt{3}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{5}{9\sqrt{3}}\right)}$$

Notes

* Area of Parallelogram

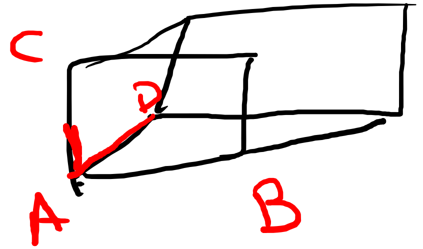


$$A = |\vec{AB} \times \vec{AC}|$$

* Area of triangle.

$$A = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

* Volume of Parallelepiped



$$V = \vec{AD} \cdot (\vec{AB} \times \vec{AC})$$

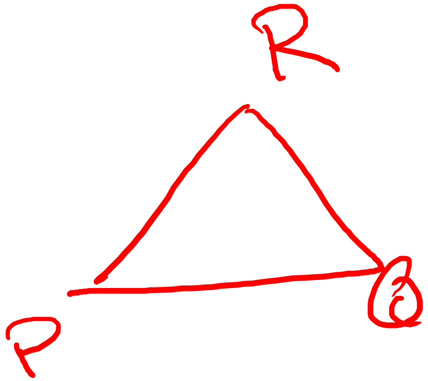
$$= C \cdot (a \times b) = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \checkmark$$

Note if $\vec{c} \cdot (\vec{a} \times \vec{b}) = 0$

that mean V of Parrel-Pied $= 0$

that means the $\vec{c}, \vec{a}, \vec{b}$ lies on the same plane.

2. Let $P=(2,1,5), Q=(-1,3,4), R=(3,0,6)$ be points in the space. Find the area of the triangle PQR.



$$\overrightarrow{PR} = R - P = \langle 1, -1, 1 \rangle$$

$$\overrightarrow{PQ} = Q - P = \langle -3, 2, -1 \rangle$$

$$A = \frac{1}{2} \left| \overrightarrow{PR} \times \overrightarrow{PQ} \right|$$

$$\overrightarrow{PR} \times \overrightarrow{PQ} =$$

$$\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ -3 & 2 & -1 \end{vmatrix} \end{array}$$

$$= (1-2)i - (\overset{2}{-1+3})j + (2-3)k$$

$$= -i - 2j - k$$

$$\left| \overrightarrow{PR} \times \overrightarrow{PQ} \right| = \sqrt{(-1)^2 + (-2)^2 + (-1)^2} = \sqrt{6}$$

$$\boxed{A = \frac{1}{2}\sqrt{6}}$$

3. Let $P = (3,0,1), Q = (-1,2,5), R = (5,1,-1), S = (0,4,2)$ be points in the space. Find the volume of the parallelepiped with adjacent edges PQ, PR, PS.

$V = \overrightarrow{PQ} \cdot (\overrightarrow{PR} \times \overrightarrow{PS})$

$\overrightarrow{PQ} = Q - P = \langle -4, 2, 4 \rangle$

$\overrightarrow{PR} = R - P = \langle 2, 1, -2 \rangle$

$\overrightarrow{PS} = S - P = \langle -3, 4, 1 \rangle$

$$V = \begin{array}{ccc} + & - & + \\ \end{array}$$
$$V = \begin{vmatrix} -4 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix}$$

$= -4(\overset{9}{1+8}) - 2(\overset{-4}{2-6}) + 4(\overset{11}{8+3})$

$= -36 + 8 \overset{52}{+44} = \boxed{16}$

4. Determine whether the points $A = (1,3,2), B = (3,-1,6), C = (5,2,0), D = (3,6,-4)$ lie in the same plane.

V Parallepiped = 0

$\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$

$V = \overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$

so they are in the same plane.