Revision

distance between two points

(x_1, y_1) (x_2, y_2)

* $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex $(2, 1)$ $(3, 4)$

$$d = \sqrt{(3-2)^2 + (4-1)^2}$$

$$= \sqrt{1 + 9} = \sqrt{10}$$

(x_1, y_1, z_1) (x_2, y_2, z_2)

m slope (x_1, y_1)

Ex Find the equation if the slope $= 2$ and pass to the point $(3, 2)$

Sol

$$y = m(x - x_1) + y_1$$

$$y = 2(x - 3) + 2$$

$$= 2x - 6 + 2$$

$$y = 2x - 4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Note

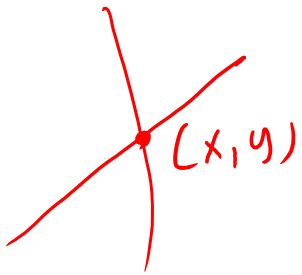
$y = m_1 x + b_1$

$y = m_2 x + b_2$

① $m_1 \neq m_2$ has one solution

$x =$

$y =$



$\Rightarrow$

$3x + 4y = 2$

$6x + 3y = 4$

$4y = -3x + 2$

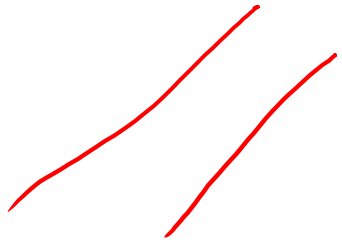
$y = \frac{-3}{4}x + \frac{1}{2}$

m

② $m_1 = m_2$

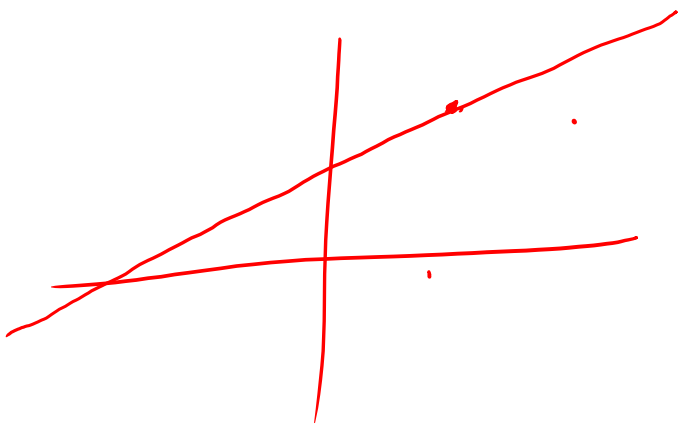
the equations have no solution answer

two lines parallel



③ $b_1 = b_2$

have infinite answers



$y = 2x + 3$

$-y = 4x + 3$

$x = 0$

$y = 3$

Functions

$\frac{1}{2}x + 4$

$x^2 - 1 = 0$

$x^2 = 1$

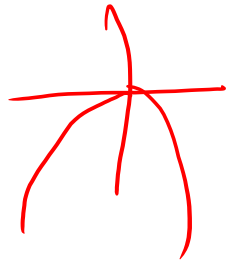
$x = \pm 1$

1. Linear equation

$y = ax + b$

D , Range R $(-\infty, \infty)$

$y = -x^2$



Range $(-\infty, 0)$

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2. Quadratic eq

$y = x^2$

$D = (-\infty, \infty)$

Range $[0, \infty)$

$y = x^2 - 1$

Domain $(-\infty, \infty)$

Range $[-1, \infty]$

$[1, 5)$

$[1, 5)$

$(1, 5]$

$(1, 5)$

$x^2 = 0$

$x = 0$

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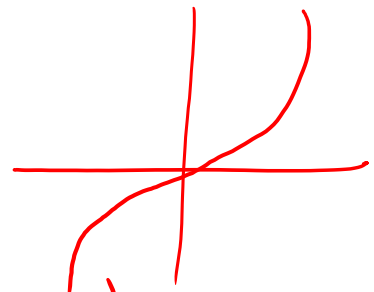
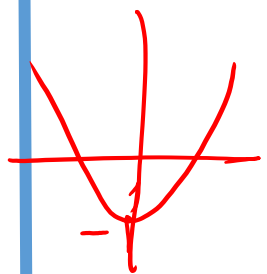
3. Cubic eq



$y = x^3$

D , Range $= (-\infty, \infty)$

R



* Rational fn

$$y = \frac{Q(x)}{R(x)}$$

Domain

$R(x) \neq 0$

$R - \{ \}$

Ex $\dfrac{x^2+1}{x-1}$

$x - 1 = 0$

$x = 1$

Domain $R - \{1\}$

Ex $y = \dfrac{x^2+1}{x^2-1}$

Domain $R - \{1, -1\}$

$x^2 - 1 = (x+1)(x-1)$

$x^3 - 1 = (x-1)(x^2+x+1)$

* Log fn

$\log_{10} x = \ln x$

ln fn

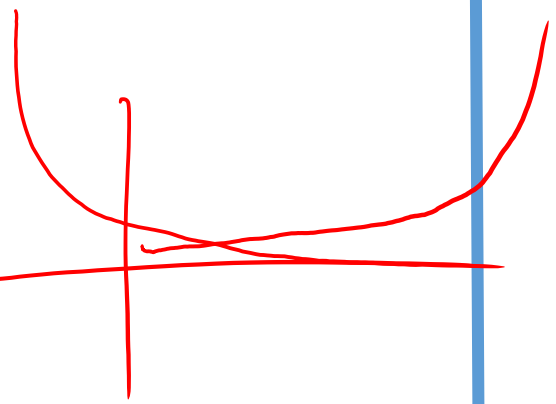
Domain $(0, \infty)$

Range $(-\infty, \infty)$

* $e^x = a^x$

Domain $(-\infty, \infty)$

Range $(0, \infty)$



$a^x = e^x$

$e = 2.718281828 - - - -$

2^x

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2^2}$

* Root fn

$\sqrt{-1} = i$ *

Ex * $y = \sqrt{x+1}$

$x+1 = 0$

$x = -1$

Domain $[-1, \infty)$

Range $[0, \infty)$ $\neq$

* $y = \sqrt{x^2+1}$

Domain $(-\infty, \infty)$

* $y = \sqrt{1-x}$

$1-x = 0$

$x = 1$

Domain $(-\infty, \cdots 0, 1]$

* $y = \sqrt{1-x^2}$

$1-x^2 = 0$

$x^2 = 1$

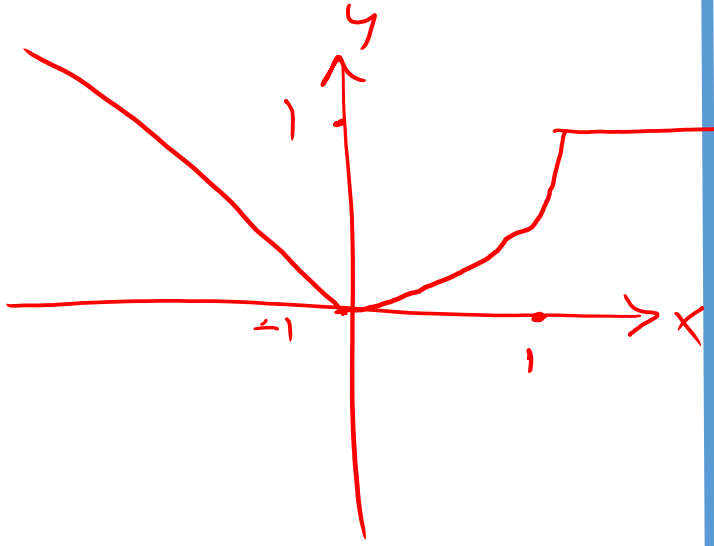
$x = \pm 1$

Domain $[-1, 1]$

x	-3	-2	-1	0	1	2	3
y							

$$y = \frac{\sqrt{x-1}}{x+2}$$

$$y = \begin{cases} -x & x < 0 \\ x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$



even & odd function

$f(-x) = f(x)$ even fn

$f(-x) = -f(x)$ odd fn

$f(x) = x^5 - x^3 - x$

$y = (-x)^5 - (-x)^3 - (-x)$

$= -x^5 + x^3 + x$

$= -(x^5 - x^3 - x)$

odd

Floor fn $\lfloor x \rfloor$ $\lfloor 2.3 \rfloor = \lfloor 2 \rfloor$

$\lfloor 2.7 \rfloor = 2$

Ceiling fn $\lceil x \rceil$ $\lfloor 4.5 \rfloor = 4$

$\lceil 2.3 \rceil = 3$ $\lceil 4.5 \rceil = 5$

$f(x) = x^4 - x^2$

$f(-x) = (-x)^4 - (-x)^2$

$= x^4 - x^2$ even

$f(x) = x^3 - x^2$

$f(-x) = (-x)^3 - (-x)^2$

$= -x^3 - x^2$

$= -(x^3 + x^2)$

neither even nor odd

Ex $y = 1 - \cos x$ even

$\cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$y = \sec x \tan x$

$= \sec(-x) \tan(-x)$

$= \sec x \, (-\tan x)$

$= -\sec x \tan x$

odd

$y = x - \cos x$

$y = (-x) - \cos(-x)$

$= -x - \cos x$

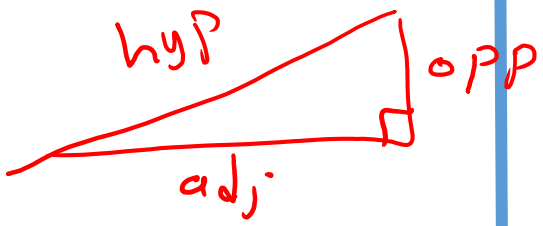
$= -(x + \cos x)$

Neither even nor odd

$\sin x = \frac{opp}{hyp}$

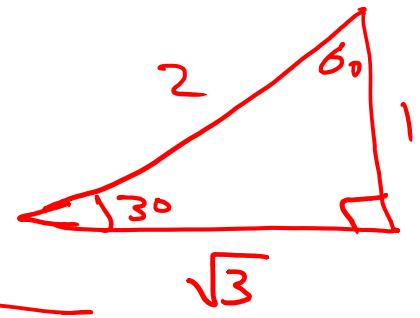
$\cos x = \frac{adj}{hyp}$

$\tan x = \frac{opp}{adj}$



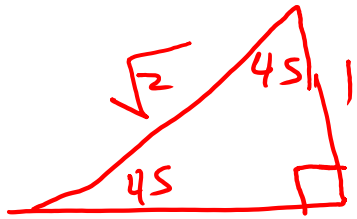
$\sin 30 = \frac{1}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$



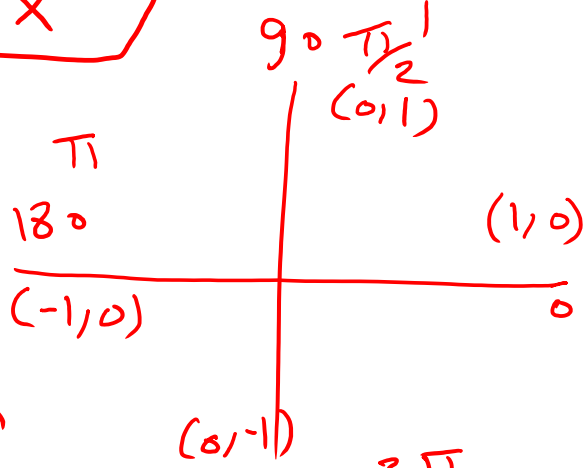
$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$



$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{1}{\tan x}$



$$\sin^2 x + \cos^2 x = 1$$

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

Combining fn

Composite fn

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

Ex $f(x) = \sqrt{x}$ $\quad g(x) = x+1$

$(f \circ g)(x) = f(g(x))$

$= f(x+1)$

$= \sqrt{x+1}$

$(g \circ f)(x) = g(f(x))$

$= g(\sqrt{x})$

$= \sqrt{x}+1$

$f(x) = \frac{1}{x}$ $\quad g(x) = \frac{1}{\sqrt{x+2}}$

Sol

$(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{\sqrt{x+2}}\right)$

$= 1 \div \frac{1}{\sqrt{x+2}} = 1 \times \frac{\sqrt{x+2}}{1}$

$= \sqrt{x+2}$

$(g \circ f)(x) = g(f(x))$

$= g\left(\frac{1}{x}\right)$

$= \frac{1}{\sqrt{\frac{1}{x}+2}}$

* properties of e^x $\ln x$

* $e^x e^y = e^{x+y}$

* $\frac{e^x}{e^y} = e^{x-y}$

* $e^{\ln x} = x$

* $\ln e^x = x$

* $\ln \frac{x}{y} = \ln x - \ln y$

* $\ln xy = \ln x + \ln y$

* $2 \ln x = \ln x^2$

Express the following logarithms in term of $\ln 2$ and $\ln 3$

$$\ln 0.75 = \ln \frac{4}{3}$$

$$= \ln 4 - \ln 3$$

$$= \ln 2^2 - \ln 3$$

$$= 2 \ln 2 - \ln 3 \ \#$$

$$\ln (4/9) = \ln 4 - \ln 9$$

$$= \ln 2^2 - \ln 3^2$$

$$= 2 \ln 2 - 2 \ln 3$$

$$\ln \sqrt[3]{9} = \ln 9^{\frac{1}{3}}$$

$$= \frac{1}{3} \ln 3^2 = \frac{2}{3} \ln 3$$

*

$$= \ln \sqrt[3]{3^2} = \ln 3^{\frac{2}{3}}$$

$$= \frac{2}{3} \ln 3$$

$$\log_{10} x = \ln x$$

$$\log_3 x$$

$$\sqrt[b]{x^a} = x^{\frac{a}{b}}$$

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inverse fun

$$y = x + 1$$

sol

$y = x + 1$

$x = y - 1$ # replace between x and y

$$y^{-1} = f^{-1} = x - 1$$

r

$y = \sqrt{x^2 - 1}$

Sol

$y^2 = x^2 - 1$

$y^2 + 1 = x^2$

$x = \sqrt{y^2 + 1}$

$y = f^{-1} = \sqrt{x^2 + 1}$

Domain $(-1, \infty)$

Range

Range $(-\infty, \infty)$

Domain

$y = e^x$

Sol

$\ln y = \ln e^x$

$\ln y = x$

$y^{-1} = \ln x$

$y = \ln x$

$e^y = e^{\ln x}$

$e^y = x$

$y = e^x$

$\sin X \longrightarrow$ Domain $(-\infty, \infty)$

$\cos X \longrightarrow$ Domain $(-\infty, \infty)$

Range $= [-1, 1]$

$y = 2\sin(3X + \pi) - 1$

Domain $(-\infty, \infty)$

Range $[-1, 1]$, $[-2, 2]$ -1

Range $[-3, 1]$ ✗

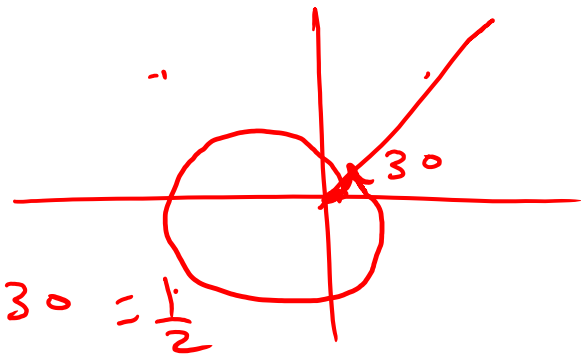
$y = 1 - \cos(X)$

D $(-\infty, \infty)$

Range

$1 - [-1, 1]$

Range $[2, 0]$



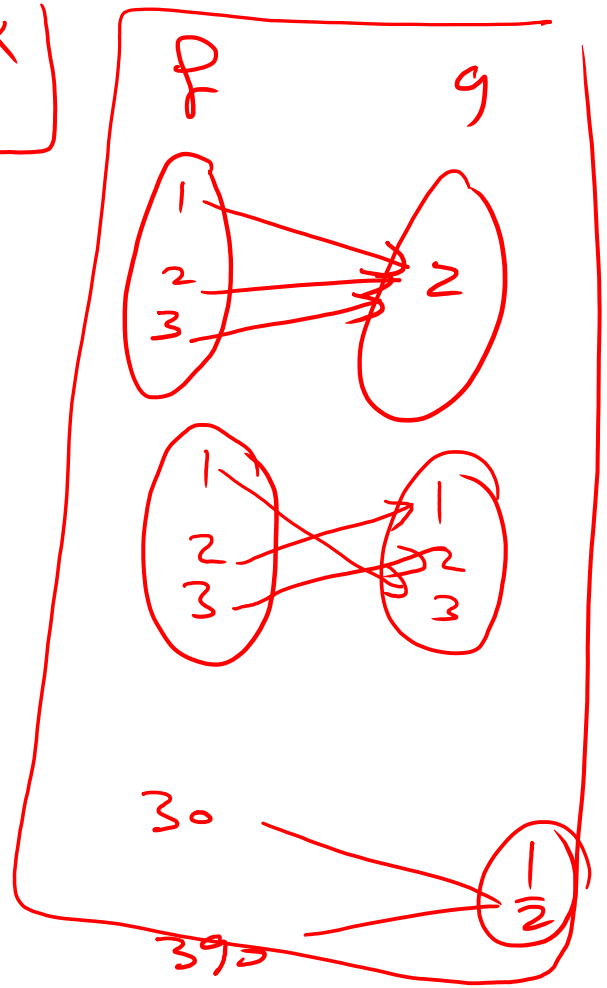
$\sin 30 = \frac{1}{2}$

$\sin(30 + 360) = \sin 390$

$= \frac{1}{2}$

$\sin \longrightarrow \sin^{-1} x$

arc sin x



$y = \sin x$

$\sin^{-1}$ $\sin^{-1}$

$x = \sin^{-1} y$

$y^{-1} = \sin^{-1} x$

arc sin x

Limits

$$\lim_{x \to 1} x+1 = 1+1 = 2$$

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{(x+1)(x-1)}{x-1} =$$

$$\lim_{x \to 1} x+1 = 1+1 = 2$$

$$\lim_{x\to 2} \frac{x-3}{x^2-4} \neq \lim_{x\to 2} \frac{(x-2)-1}{(x-2)(x+2)}$$

$$= \frac{-1}{0} = \infty$$

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x+2)} - \frac{1}{(x-2)(x+2)}$$

$$\lim_{x\to 2} \frac{1}{x+2} - \frac{1}{(x-2)(x+2)}$$

$$\lim_{x\to 2} \frac{1}{x+2}\left(1 - \frac{1}{x-2}\right) \quad \#$$

$$\frac{1}{0} = \infty$$

$$\lim_{x\to 2} \frac{x+2}{\sqrt{x^2+5}-3} = \frac{4}{0}$$

$$\lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$

$$\lim_{x\to -2} \frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{x^2+5-9} \quad \leftarrow x^2-4$$

$$= \lim_{x\to -2} \frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{(x+2)(x-2)}$$

$$= \frac{3+3}{-2-2} = \frac{6}{-4}$$

$$\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$$

$$\lim_{x\to 0} -x^2 = 0$$

$$\lim_{x\to 0} x^2 = 0$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$$

$$\lim_{x\to \infty} \frac{1}{x} = 0$$

$$\lim_{x\to \infty} \frac{C}{x^n} = 0$$

$$\lim_{x\to 0} \frac{\sin 3x}{2x} = \frac{3}{2}$$

$$\lim_{x\to \infty} \frac{x^2+3x+1}{x^3+x+1}$$

$$\lim_{x\to 0} x^2 \sin\frac{1}{x}$$

Sol

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-x^2 \le x^2 \sin\frac{1}{x} \le x^2$$

$$\lim_{x\to \infty} \frac{\frac{x^2}{x^3}+\frac{3x}{x^3}+\frac{1}{x^3}}{\frac{x^3}{x^3}+\frac{x}{x^3}+\frac{1}{x^3}} =$$

$$\lim_{x\to \infty} \frac{\frac{1}{x}+\frac{3}{x^2}+\frac{1}{x^3}}{1+\frac{1}{x^2}+\frac{1}{x^3}} = \frac{0+0+0}{1+0+0} = 0$$

$$\lim_{x\to\infty} \frac{x^2+x+1}{3x^2+2x+1}$$

$$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}{3\frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{1}{x^2}}{3+\frac{2}{x}+\frac{1}{x^2}}$$

$$= \frac{1+0+0}{3+0+0} = \frac{1}{3}$$

$\infty + \infty = \infty$

$\infty - \infty = \infty$

$\infty \cdot \infty = \infty$

$\frac{1}{\infty} = 0$

$\frac{\infty}{\infty}$ undefind

Continuity

$f(x)$ at $x = b$

① $f(b)$

② $\lim_{x\to b} f(x)$

③ $\lim_{x\to b} f(x) = f(b)$

$$f(x) = \begin{cases} x+1 & x \leq 0 \\ x^2+1 & x > 0 \end{cases}$$

Study the Continuous of this function

$f(0) = 0 + 1 = 1$

$\lim_{x \to 0^-} x+1 = 0+1 = 1$

$\lim_{x \to 0^+} x^2+1 = 0+1 = 1$

this function is Continuous

$$f(x) = \begin{cases} x^2+2 & x \neq 2 \\ 6 & x = 2 \end{cases}$$

$f(2) = 6$

$\lim_{x \to 2} x^2+2 = 4+2 = 6$

Continuous

$f(x) = x^2$

$f(x+h) = (x+h)^2$

$= x^2 + 2xh + h^2$

$f(x) = 3x^2$

rate of change

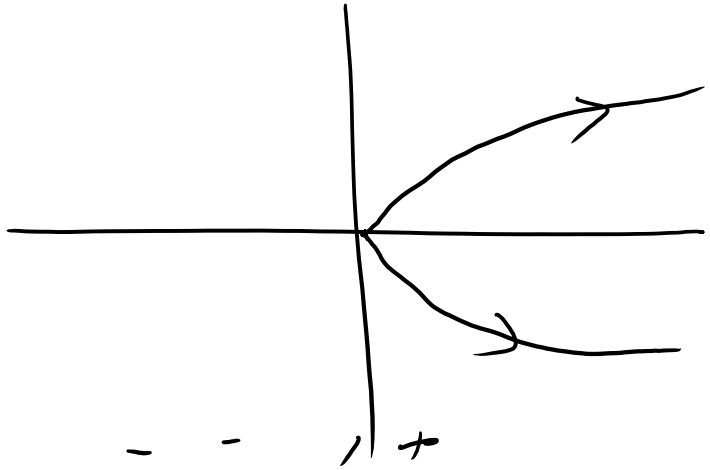
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

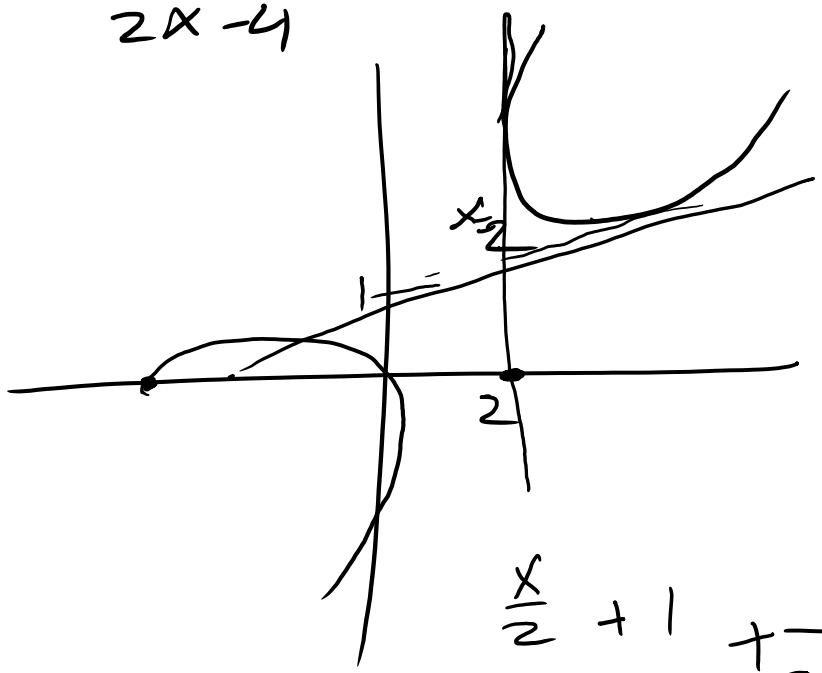
$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$\lim_{h \to 0} 2x + h = 2x + 0 = 2x$$

$$f(x) = 1 - \frac{1}{x+2}$$



$$y = \frac{x^2-3}{2x-4}$$



$$\frac{\frac{x}{2}+1}{} + \frac{1}{2x-4}$$

$$\lim_{x \to \infty} f(x) = L$$

$$|f(x) - L| < \varepsilon$$