

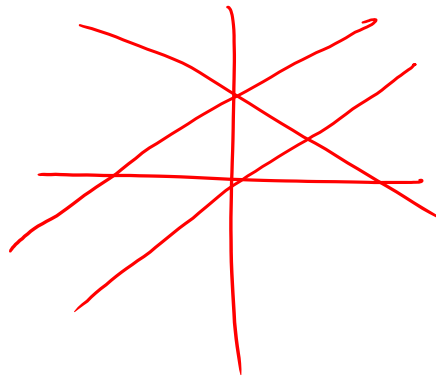
# Lecture 1

## Linear Equation in Linear Algebra

### 1.1 system of linear Equations

equation  $y = x$  linear equation

$$y = x + 1$$
$$x + 2 = 3$$



~~$$x + 2y = 3$$~~~~$$-x + y = 4$$~~

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$$3y = 7$$

$y = \frac{7}{3}$

$x, y$

### \* Linear equation

$$a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n = b$$

coefficients

$x_1 \quad x \quad x_3$

$$2x + 3 = y$$

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### system of linear equation

1] no solution  $x + y = 5, 2x - y = 1$

$x = 2$   
 $y = 3$

2] exactly one solution

3] infinity many solutions

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no solution (inconsistent)  
exactly 1 or infinity (consistent)

no solution

$$-4x + 10y = 6$$

$$2x - 5y = 3 \quad \times 2$$

$$\begin{array}{r} \hline -4x + 10y = 6 \\ 4x - 10y = 6 \\ \hline 0 = 12 \end{array}$$

no solution

$$\boxed{0 = 0}$$

EX  $y = -3x + 9$

$$y = -3x - 7$$

$0 = 16$  no solution (inconsistent)

infinity many solution

$$-6x + 4y = 2$$

$$3x - 2y = 1$$

$$\begin{array}{r} \hline -6x + 4y = 2 \\ \hline \end{array}$$

$$6x - 4y = 2$$

$$\begin{array}{r} \hline 0 = 4 \quad \text{no solution} \\ \hline \end{array}$$

$$x + y = 1$$

$$2x + 2y = 2$$

$$\begin{array}{r} \hline -2x - 2y = -2 \\ \hline \end{array}$$

$$2x + 2y = 2$$

$$\boxed{0 = 0}$$

infinity many solutions

# Matrix

row  $a_{ij}$   
col

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \leftarrow \text{Row} \quad \text{EX} \\ \uparrow \text{Column} \\ 2 \times 3$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix}$$

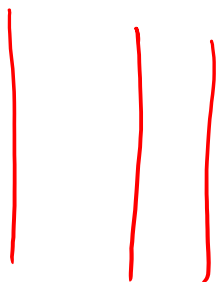
row  $\rightarrow$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} 3 \times 2$$

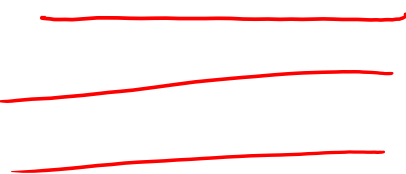
$$= \begin{bmatrix} 1+2 & 2+1 \\ 3+3 & 4+3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ 6 & 7 \end{bmatrix}$$

Column



row



$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix} 3 \times 3$$

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$$\text{EX} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow R_1 + R_3 \Rightarrow R_3$$

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 3 & 5 \end{bmatrix} \Rightarrow \underline{-3R_1 + R_2} \Rightarrow R_2$$

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -4 & -8 \end{bmatrix}$$

augmented matrix

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}_{3 \times 3}$$

✓ Coefficient Matrix

$$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ -4 & 5 & 9 & -9 \end{bmatrix}_{3 \times 4}$$

✓ Augmented Matrix

Ex 1 Solve the given system of equation

$$\begin{aligned}
 x_1 - 2x_2 + x_3 &= 0 \\
 2x_2 - 8x_3 &= 8 \\
 -4x_1 + 5x_2 + 9x_3 &= -9
 \end{aligned}$$

SOL  
 ← Elimination Method (باز)

$$\begin{array}{l}
 R_1 \\
 R_2 \\
 R_3
 \end{array}
 \left[ \begin{array}{cccc}
 1 & -2 & 1 & 0 \\
 0 & 2 & -8 & 8 \\
 -4 & 5 & 9 & -9
 \end{array} \right]$$

$$4R_1 + R_3 \Rightarrow R_3$$

$$\left[ \begin{array}{cccc}
 1 & -2 & 1 & 0 \\
 0 & 2 & -8 & 8 \\
 0 & -3 & 13 & -9
 \end{array} \right]$$

$$R_2/2 \rightarrow R_2$$

$$\left[ \begin{array}{cccc}
 1 & -2 & 1 & 0 \\
 0 & 1 & -4 & 4 \\
 0 & -3 & 13 & -9
 \end{array} \right]$$

$$3R_2 + R_3 \rightarrow R_3$$

$$\begin{array}{c}
 x_1 \quad x_2 \quad x_3 \\
 \left[ \begin{array}{cccc}
 1 & -2 & 1 & 0 \\
 0 & 1 & -4 & 4 \\
 0 & 0 & 1 & 3
 \end{array} \right]
 \end{array}$$

$$x_1 - 2x_2 + x_3 = 0$$

$$x_2 - 4x_3 = 4$$

$$x_3 = 3$$

$$x_2 - 4 \cdot 3 = 4$$

$$x_2 = 4 + 12 = 16$$

$$x_1 - 2x_2 + x_3 = 0$$

$$x_3 = 3$$

$$x_2 = 16$$

$$x_1 - 32 + 3 = 0$$

$$x_1 - 29 = 0$$

$$\boxed{x_1 = 29}$$



كورس خانة

Course Khana

05528598792

# Elimination

$$\left[ \begin{array}{l} a_1x_1 + a_2x_2 + a_3x_3 = D_1 \\ b_1x_1 + b_2x_2 + b_3x_3 = D_2 \\ c_1x_1 + c_2x_2 + c_3x_3 = D_3 \end{array} \right]$$

$$\left[ \begin{array}{cccc} 5 & 8 & -4 & 3 \\ 6 & 1 & -5 & 4 \\ 4 & 7 & -2 & 2 \end{array} \right] \begin{array}{l} -4R_1 + 5R_3 \Rightarrow R_3 \\ -6R_1 + 5R_2 \Rightarrow R_2 \end{array}$$

$$\left[ \begin{array}{cccc} 5 & 8 & -4 & 3 \\ 0 & 49 & & \\ 0 & 12 & 0 & 0 \end{array} \right] \begin{array}{l} \curvearrowright x_1 \\ \curvearrowright x_2 \\ x_3 \end{array}$$