§1.2: Basic Ideas and Terminology.

المعادلة التفاضلية

Def'n: A Differential Equation is an equation involving derivatives of an unknown function.

$y'' + y = x$

$y''' + xy' + y = 0$

Ex:

(a) $\dfrac{dy}{dx} + y = x^2$ (y')

(b) $\dfrac{d^2y}{dx^2} = -k^2 \cdot y \Rightarrow y'' = -k^2 y$

(c) $\dfrac{d^3y}{dx^3} + \left(\dfrac{d^2y}{dx^2}\right)^5 + \cos x = 0 \Rightarrow y''' + (y'')^5 + \cos x = 0$

(d) $\sin\left(\dfrac{dy}{dx}\right) + \arctan y = 1$ $\quad \boxed{\sin y' + \tan^{-1} y = 1}$

عادية

(e) $F_{xx} + F_{yy} - F_x = e^x + x \cdot \sin y$ $\quad \dfrac{\partial^2 y}{\partial^2 x}$

ليس عادية
تفاضلية

In (a), (b), (c), and (d) the unknown fn. is y and there is ONLY ONE independent variable x.

$\Rightarrow$ We call these DE, Ordinary Differential Equations.

(e) is a Partial Diff. Eq. (or PDE), because the unknown fn. F is differentiated w.r.t. two indep. variables x and y.

* In this course, DE means Ordinary Diff. Eqn.

Def'n: The order of the highest derivative is called the $\boxed{\text{ORDER}}$ of the DE.

تعرف: اكبر مشتقة
المعادلة

Ex: (previous)

(a) Order is 1
(b) Order is 2
(c) Third order
(d) First Order
(e) Second Order.

$$y''' + y'' + x^2 + 1 = 0$$

ما رتبتها ؟

3

$x + 3 = 4$

$\boxed{x = 2\ ?}$

$x = 1$

Ex: $\boxed{y''' + y'' - y' - y = 0}$
is a DE of order 3.

Show that $\boxed{y = e^{-x}}$ is a solution?

If $y = e^{-x}$, then $y' = -e^{-x}$
$y'' = e^{-x}$
$y''' = -e^{-x}$

$y' = e^{-x}(-1) = -e^{-x}$
$y'' = -e^{-x}(-1) = e^{-x}$
$y''' = -e^{-x}$

and therefore $y''' + y'' - y' - y =$
$= e^{-x} + (-e^{-x}) - (e^{-x}) - (-e^{-x})$
$= 0$ ✓

$y = e^{u} \rightarrow y' = e^{u} \cdot u'$
$y = e^{x^2} \rightarrow y' = e^{x^2} \cdot 2x$

Therefore, $y = e^{-x}$ is a solution.

Actually, $y = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x} + C_3 \cdot e^{x}$
is a solution for any C_1, C_2, C_3.
[Check!]

$y' = 5 \rightarrow$ $\boxed{\begin{array}{l} y = 5x \\ y = 5x+2 \\ y = 5x+1 \end{array}}$ $\qquad \boxed{y = 5x + c}$

Partial sol حلول خاصة

الحل العام General sol

Def'n: Given a Differential Equation of order $\boxed{n}$ (The highest derivative in the DE is $y^{(n)}$)

- Any fn. $y = f(x)$ that satisfies the DE is called a Particular Solution.

- If a solution $y = g(x)$ is containing n constants $\boxed{C_1, \dots, C_n}$ and if every solution of the DE can be obtained by assigning values to $\boxed{C_1, C_2, \dots, C_n}$,

تعريف then $y = g(x)$ is called the General Solution.

$y'' + 1 = 5 \rightarrow y = \{C_1, C_2\}$ ثابتين

Ex: $\boxed{y'' = -g}$ $\Rightarrow$ order $= 2$

General solution: $y = -\frac{1}{2} g \cdot t^2 + C_1 t + C_2$ — contains two constants C_1 and C_2. → حل عام

A (particular) sol'n: $y = -\frac{1}{2} g t^2 + 20t + 100$

Another sol'n: $y = -\frac{1}{2} g t^2$

$y = -\frac{1}{2} g t^2 + 5t + 10$ حل خاص

تعريف

$y''' + y = 1$

$y = \{C_1, C_2, C_3\}$

ثلاث ثوابت

Ex: $y''' + y'' - y' - y = 0$. $\quad 3$

حل عام General Solution: $y = C_1 e^x + C_2 \cdot x \cdot e^x + C_3 e^{-x}$

A (particular) sol'n: $y = e^{-x}$

Another sol'n: $y = 2 \cdot e^x - 3x \cdot e^x - 7 \cdot e^{-x}$ حل خاص

$y = 2e^x + 5xe^x + 3e^{-x}$

$C_1 = 2, \; C_2 = 5, \; C_3 = 3$

* Some DE don't have General solutions.

EX: $(y')^2 + (y-1)^2 = 0$.

$\Rightarrow y' = 0$ and $y - 1 = 0$
$y = 1$.

Similar to $a^2 + b^2 = 0$
$\Rightarrow a = 0$
$b = 0$.

So, the only sol'n is the constant fn. $y = 1$.

The Order is 1, but there is no C in the only sol'n.

$\Rightarrow$ No Gen. Sol'n.

Def'n: ~~Given~~ A DE of order n, with n conditions:

$y(x_0) = y_0,\ y'(x_0) = y_1,\ y''(x_0) = y_2, \ldots,\ y^{(n-1)}(x_0) = y_{n-1}$

Initial Conditions.

is called an $\boxed{\text{Initial Value Problem (IVP)}}$.

EX: $y'' = -g,\ y(0) = 100,\ y'(0) = -20$.
is an IVP

EX: $y''' + y'' - y' - y = 0$; $y(2) = 5,\ y'(2) = -1,\ y''(2) = 0$.

(How many Initial Conditions do we need?
A = 3

$\boxed{y' = 5}$, $\boxed{y(1) = 7}$

$y = 5x + C$

$5 + C = 7$

$\Rightarrow C = 2$

$\Rightarrow \boxed{y = 5x + 2}$

Q: How do we solve an IVP?

A: First find all solutions of the DE:

$y = C_1 \cdot e^x + C_2 \cdot x \cdot e^x + C_3 \cdot e^{-x}$

then find the sol'n that satisfies the Initial conditions.

For example $y(2) = 5 \Rightarrow C_1 e^2 + C_2 \cdot 2e^2 + C_3 e^{-2} = 5$

$y'(2) = -1 \Rightarrow C_1 e^2 + 3 \cdot C_2 \cdot e^2 - C_3 e^{-2} = -1$

Find C_1, C_2, C_3.