Lecture 1

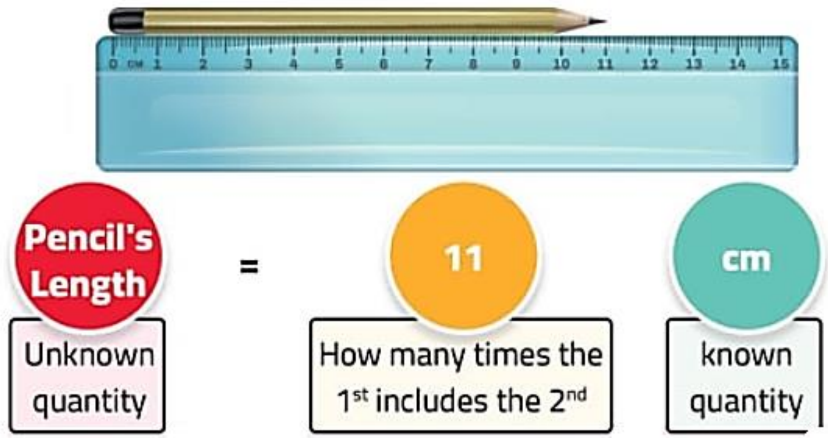
Physical Measurements, Units and Dimensional Formula

Lecture`s Outlines:

1) Physical quantities and Measuring Tools
2) Measuring Units (Systems, SI, Standard Units, Prefixes, Conversion of units, Accuracy, Precession, Uncertainty and Significant Figures)
3) Dimensional Formula

1) Physical quantities and Measuring Tools

Physical Measurement process elements are physical quantity, tool and unit.



a) Physical Quantities: Classified according to derivation

Fundamental

- Quantity that cannot be defined in terms of other
- **Ex: Length, Mass, Time**

Derived

- Quantity that is defined in terms of fundamental
- **Ex: Force, Speed, Work**

Example 1:

The fundamental physical quantities from the following are ....

(a) the length and the area
(b) the velocity and the acceleration
(c) the mass and the volume
(d) the time and the mass

Answer: d

Example 2:

The derived physical quantities from the following are ....

(a) velocity - distance - time
(b) mass - density - volume
(c) work - force - distance
(d) force - volume - density

Answer: d

b) Measuring Tools

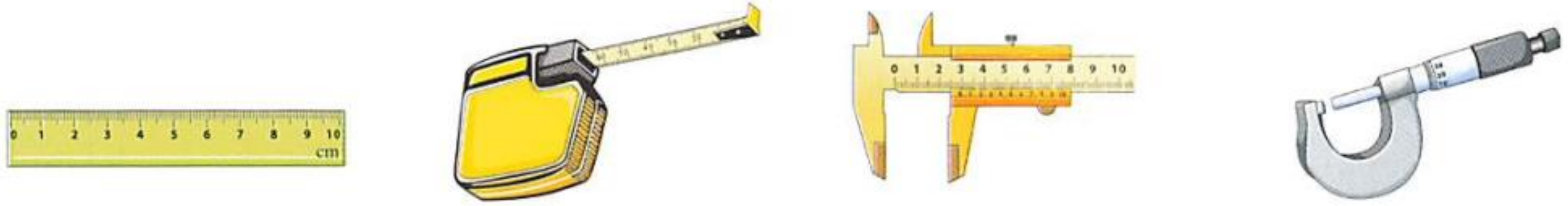
- Ruler: - Small distance
- Meter tape: - Large distance
- Vernier caliper: - Accurate - V.Small distances
- Micrometer: - V.Accurate - V.Small distances

- Roman scale: - Large objects -inaccurate
- Beam balance: - Ray Balance
- Analog scale:
- Digital balance: -Small object -accurate

- Hourglass: v.old
- Clock: Based on Conserv. Of Energy
- Stopwatch:
- Digital watch: -accurate

Example 3:

The suitable tool for measuring the length of a room is ....



(a) (b) (c) (d)

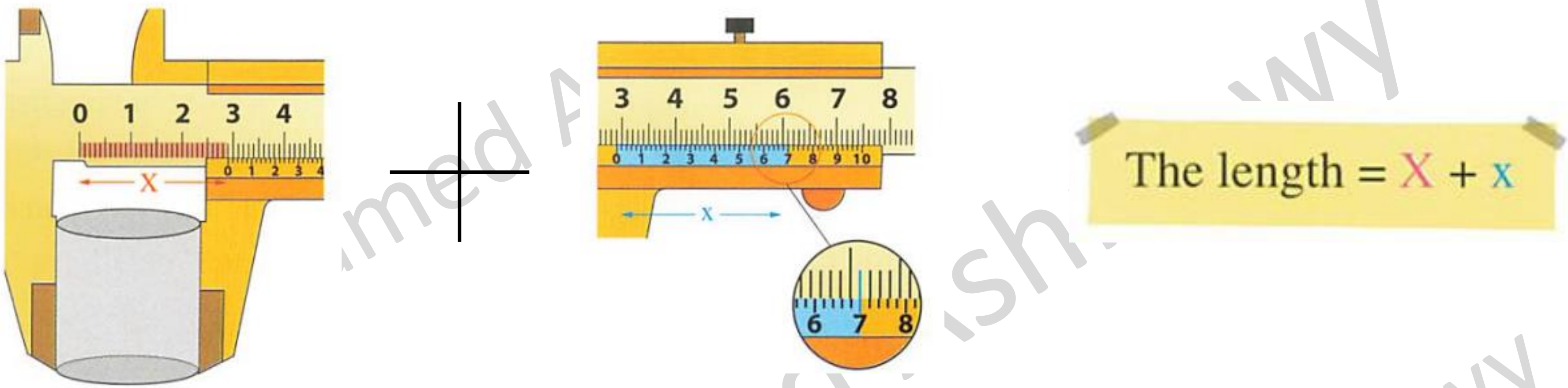
Answer: b

Example 4:

The suitable tool for measuring the mass of a golden ring is ....

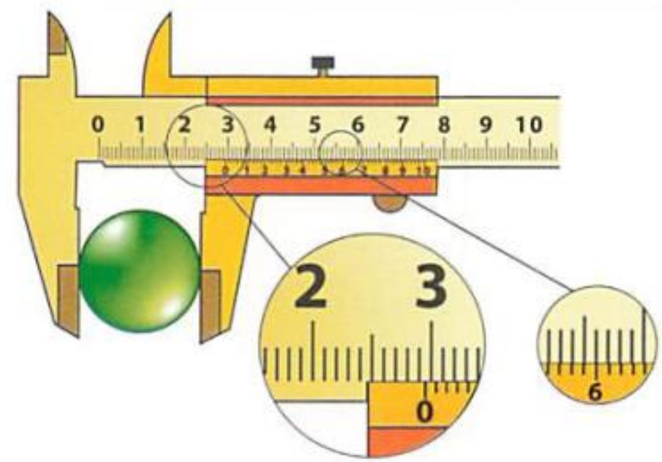
(a) (b) (c) (d)

Answer: d



Example 4 (a):

Using the opposite figure, the external diameter of the ball is ....



ⓐ 29 mm ⓑ 29.1 mm

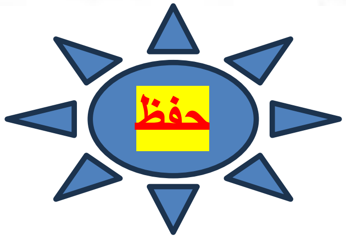
ⓒ 29.6 mm ⓓ 35 mm

Answer: c

2) Measuring Units (Systems, SI, Standard Units, Prefixes and Conversion of Units)

a) Unit Systems

The fundamental physical quantity / System of units	Units of measurement: The French system (Gaussian system) (C.G.S)	Units of measurement: The British system (F.P.S)	Units of measurement: The Metric system (M.K.S)
Length (ℓ)	Centimeter (cm)	Foot (ft)	Meter (m)
Mass (m)	Gram (g)	Pound (lb)	Kilogram (kg)
Time (t)	Second (s)	Second (s)	Second (s)

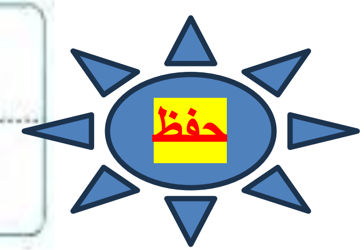


Units of Length, Mass, and Time

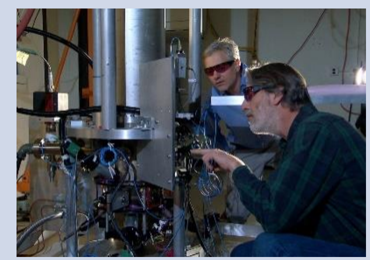
Dimension	SI	CGS	U.S. Customary Units
length	meter (m)	centimeter (cm)	foot (ft)
mass	kilogram (kg)	gram (g)	slug
time	second (s)	second (s)	second (s)

b) International System (SI) Unit

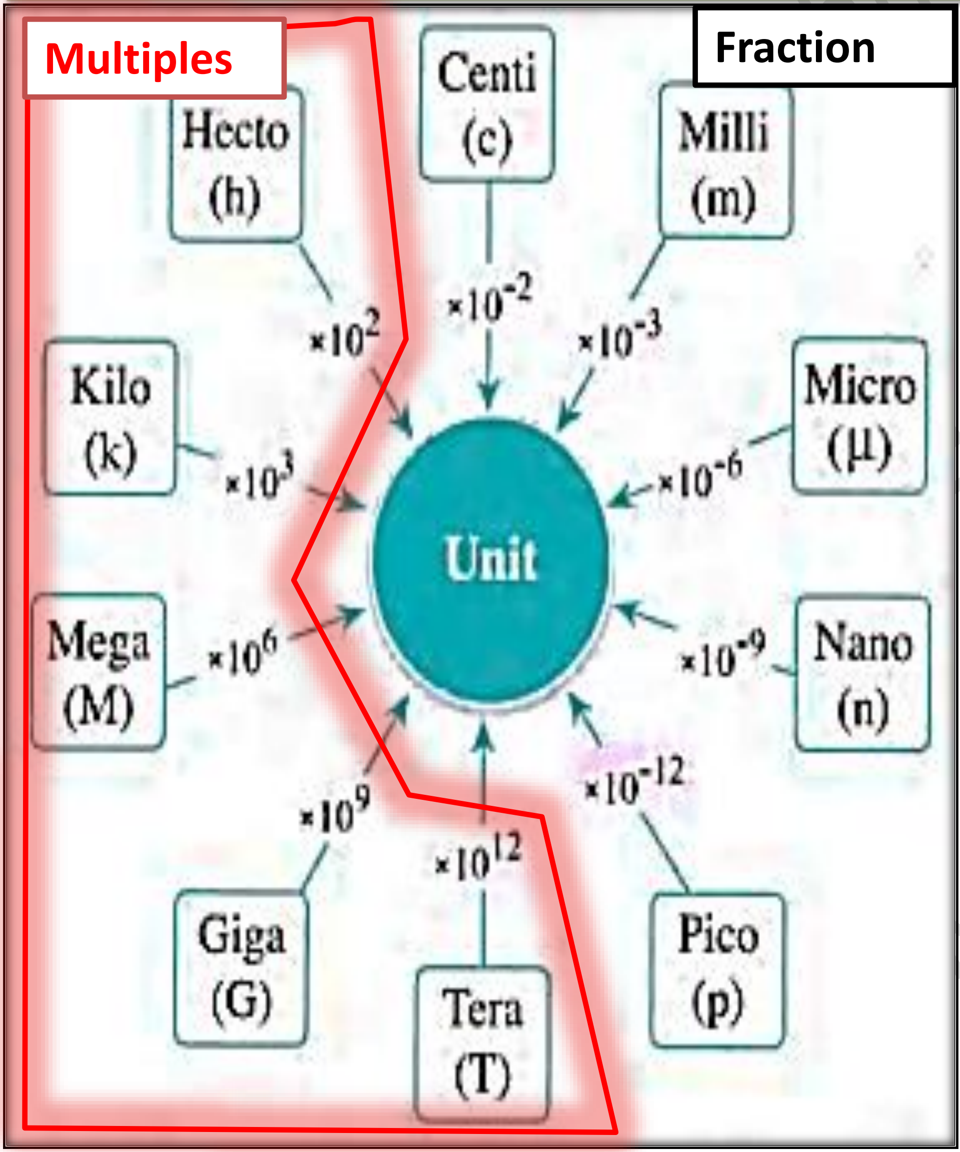
The physical quantity	The international unit
1 Length (ℓ)	Meter (m)
2 Mass (m)	Kilogram (kg)
3 Time (t)	Second (s)
4 Electric current intensity (I)	Ampere (A)
5 Absolute temperature (T)	Kelvin (K)
6 Amount of substance (n)	Mole (mol)
7 Luminous intensity (I_v)	Candela (cd)
8 Plane angle	Radian (rad)
9 Solid angle	Steradian (sr)



c) Standard Units

The Standard Length (The Standard Meter)	The Standard Mass (The Standard Kilogram)	The Standard Time (The Standard Second)
It is the distance between two engraved marks at the ends of a rod made of platinum and iridium alloy kept at 0°C. 	It is the mass of a cylinder made of platinum and iridium alloy of specific dimensions kept at 0°C. 	**Usually 1 second measured w.r.t. :** Day and night times $1\text{ Sec} = \frac{1}{24\times60\times60} = \frac{1}{86400}\text{ Day}$ **Recently**: An Atomic (Cesium) clock is used [Accurate] 
Platinum-iridium alloy is rigid, chemically inactive, and not affected by surrounding temp.		Cesium Clock usage: 1. Determination of the duration of the Earth's spin 2. Checking up on aviation and navigation. 3. Verify the journey schedule of spaceships.

d)Prefixes (Multiples and Fractions) of units in SI & Conversion of units



1) Liter (L) is volume unit of Liq. And gases ($1L = 10^{-3}\ m^3$)
2) 1 gram (g) = 10^{-3} kg
3) 1 Ton = 10^3 kg

The reason of prefixes

Fractions Examples

Multiples Examples

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in Meters		Masses in Kilograms (more precise values in parentheses)		Times in Seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron 9.11×10^{-31} kg	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom 1.67×10^{-27} kg	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day 8.64×10^{4} s
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $(3.16 \times 10^{7}$ s$)$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from Earth to the Sun	10^{23}	Mass of the Moon $(7.35 \times 10^{22}$ kg$)$	10^{17}	Age of Earth
10^{16}	Distance traveled by light in one year (a light year)	10^{25}	Mass of Earth $(5.97 \times 10^{24}$ kg$)$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way Galaxy	10^{30}	Mass of the Sun $(1.99 \times 10^{30}$ kg$)$		
10^{22}	Distance from Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way Galaxy (current upper limit)		
10^{26}	Distance from Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

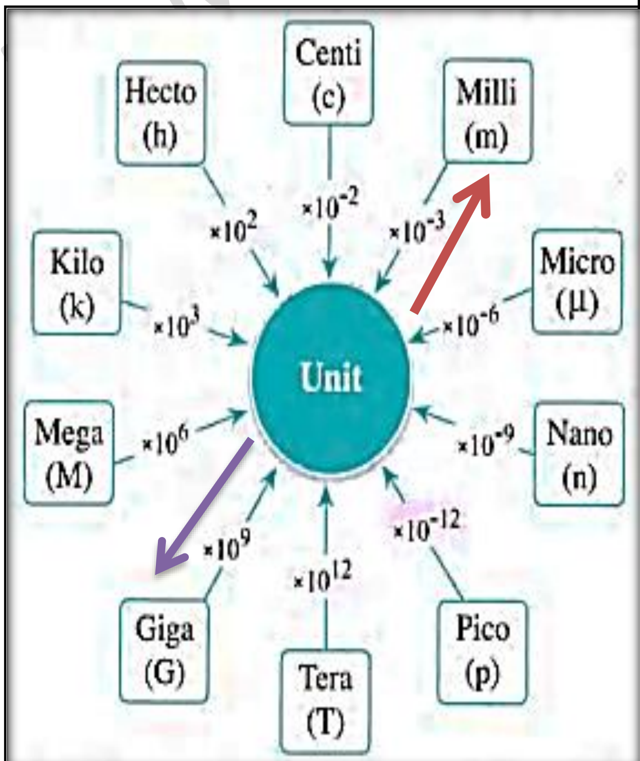
Do Not Forget

6 Rules of exponents

Rule	Example
$x^0 = 1$	$(2^0) = 1$
$x^1 = x$	$(-4)^1 = -4$
$x^{-m} = \frac{1}{x^m}$	$(3)^{-2} = \frac{1}{(3)^2} = \frac{1}{9}$
$(x^m)^n = x^{mn}$	$(2^2)^3 = (2)^{2\times 3} = (2)^6 = 64$
$(xy)^m = x^m y^m$	$(2\times 3)^2 = (2)^2 \times (3)^2 = 36$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{1}{3}\right)^2 = \frac{(1)^2}{(3)^2} = \frac{1}{9}$
$x^m x^n = x^{m+n}$	$(2)^3 \times (2)^{-2} = (2)^{3+(-2)} = (2)^1 = 2$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{(3)^4}{(3)^{-2}} = (3)^{4-(-2)} = (3)^6 = 729$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	$(8)^{\frac{1}{3}} = \sqrt[3]{8} = 2$

How to convert same quantity with different prefixes in SI Units (1st Case: units without power)

Given Number with given prefix ✖ $\dfrac{\text{1 Unit = ???? Of the **Required** Prefix (R)}}{\text{1 Unit = ???? Of the **Given** Prefix (G)}}$ = New Number with Required prefix



لإيجاد ال **Factor** اللي هو طالبه أو اللي عندك (المعطي) من الوحده الأصليه

إعكس إتجاهات الأسهم بحيث تتحرك من ال **Unit** ال المطلوبه **prefix**
وطبعا هتغير الاشاره بتاعت الأس ال **Power**

يعني تقول : هي ال SI Unit بتساوي كام Milli (m)
Giga (G)
Nano (n)
..
...
...

This method called Chain- Link conversion method: Multiply the original measurement by a conversion factor (a ratio of units that is equal to unity)

Example 5:

✻ 86.2 cm is equal to ....

(a) 8.62 m (b) 8.62×10^{-4} km (c) 0.862 mm (d) 862×10^{10} µm

Answer:

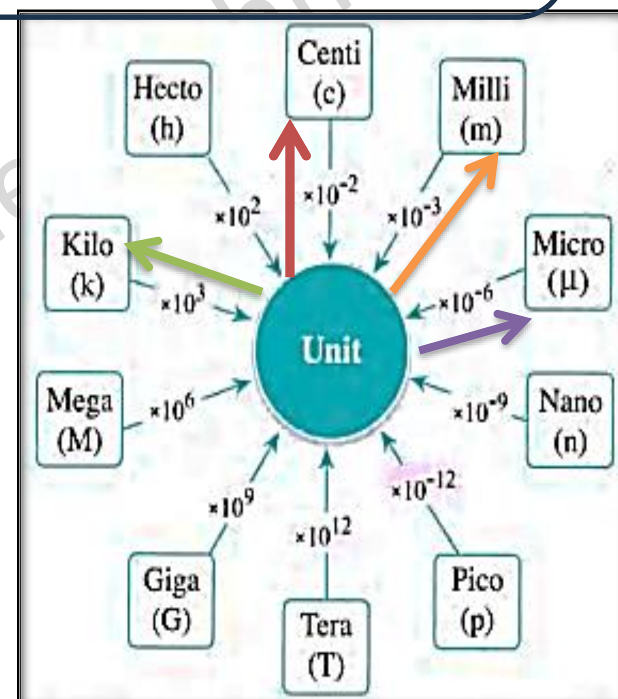
Given Number with given prefix × $\dfrac{\text{1 Unit = ???? Of the **Required** Prefix (R)}}{\text{1 Unit = ???? Of the **Given** Prefix (G)}}$ = New Number with Required prefix

a) 86.2 ~~cm~~ x ($\dfrac{1\ m}{10^2\ \not{cm}}$) = 0.862 m

b) 86.2 ~~cm~~ x ($\dfrac{10^{-3}\ km}{10^2\ \not{cm}}$) = 86.2×10^{-5} = 8.62×10^{-4} km

c) 86.2 ~~cm~~ x ($\dfrac{10^3\ mm}{10^2\ \not{cm}}$)= 862 mm

d) 86.2 ~~cm~~ x ($\dfrac{10^6 µm}{10^2\ \not{cm}}$)= 86.2×10^4 µm



Tricks of the questions

1) Take care of the units of the choices
2) **Changing the position of the decimal point**

Moving decimal point to the Right then subtract the power (RS)

Ex1: 573.2105×10^{2} ⊖ =

$5732.105 \times 10^{2-1} = 57321.05 \times 10^{1} =$

$57321.05 \times 10^{1-1} = 57321.05 \times 10^{0} =$

57321.05

Ex2: $573.2105 \times 10^{-2} =$

$5732.105 \times 10^{-2-1} = 5732.105 \times 10^{-3} =$

$57321.05 \times 10^{-4} = 573210.5 \times 10^{-5} =$

5732105.0×10^{-6}

Moving the decimal point to the left then add to the power (LA)

Ex1: 573.2105×10^{2} ⊕ =

$57.32105 \times 10^{2+1} = 57.32105 \times 10^{3} =$

$5.732105 \times 10^{3+1} = 5.732105 \times 10^{4} =$

$0.5732.105 \times 10^{5} = 0.05732105 \times 10^{6}$

Ex2: $573.2105 \times 10^{-2} =$

$57.32105 \times 10^{-2+1} = 57.32105 \times 10^{-1} =$

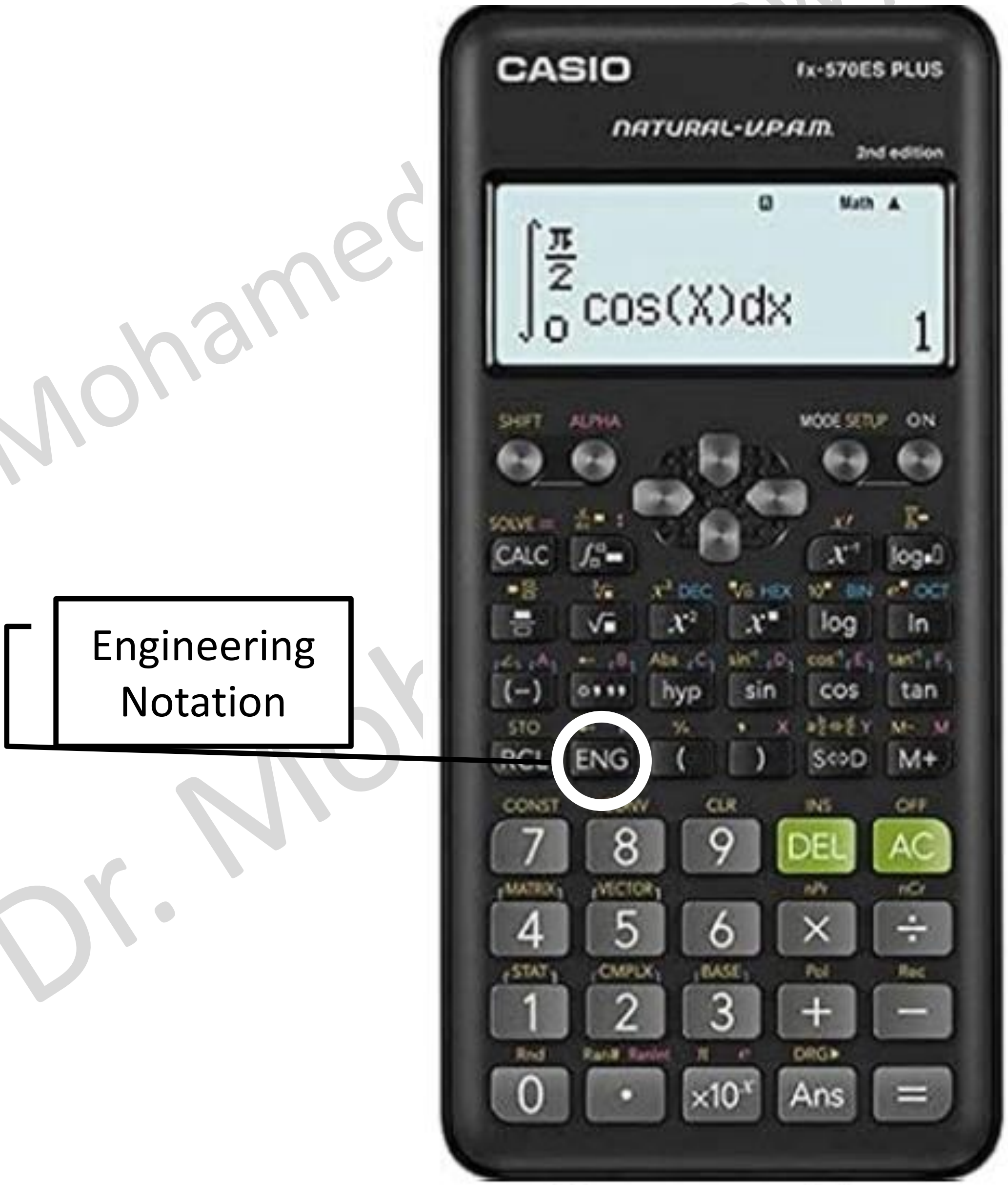
$5.732105 \times 10^{-1+1} = 5.732105 \times 10^{0} =$

$0.5732105 \times 10^{0+1} = 0.5732105 \times 10^{1}$

Rule: RS- LA

Right Subtract- Left Add

Check from “ENG” button from Calculator


Engineering Notation
CASIO
fx-570ES PLUS
NATURAL-V.P.A.M.
2nd edition
ENG

Example 6:

If the radius of the hydrogen atom is 0.053 nm, then it is equivalent to ....

(a) 0.53×10^{-10} m (b) 5.3×10^{-11} m (c) 53×10^{-12} m (d) all the previous

Answer:

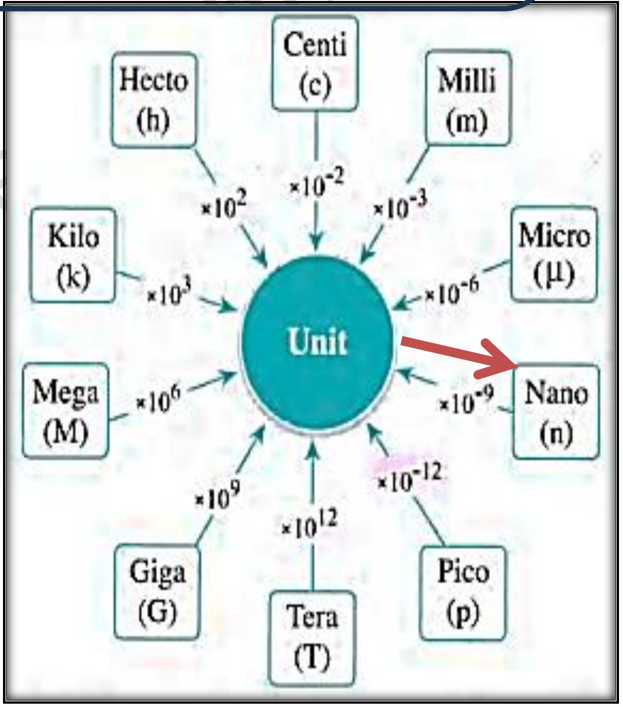
Given Number with given prefix ✖ (1 Unit = ???? Of the **Required** Prefix **(R)** / 1 Unit = ???? Of the **Given** Prefix **(G)**) = New Number with Required prefix

a) 0.053 nm x ($\frac{1\ m}{10^9\ nm}$) = 0.053 x 10^{-9}= 0.53 x 10^{-10} m

b) 0.053 nm x ($\frac{1\ m}{10^9\ nm}$) = 0.053 x 10^{-9} = 5.3 x 10^{-11} m

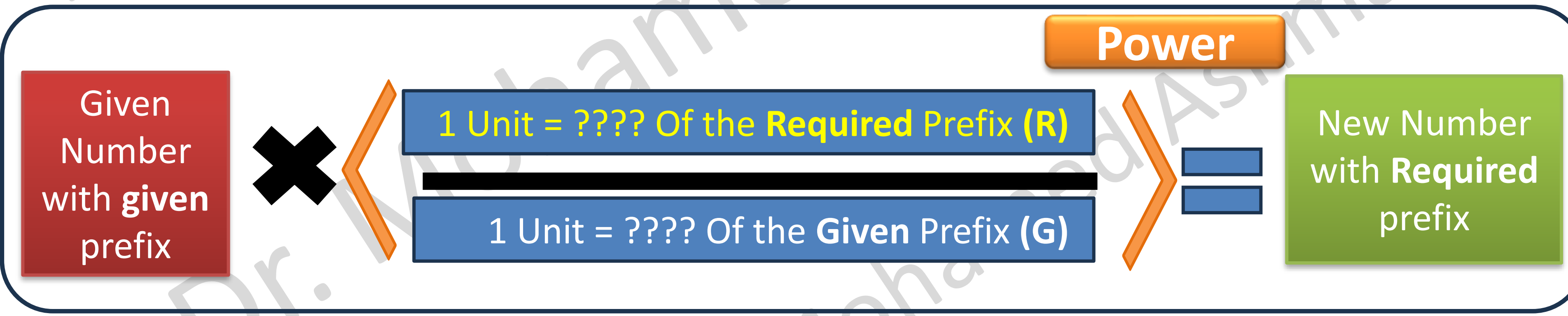
c) 0.053 nm x ($\frac{1\ m}{10^9\ nm}$) = 0.053 x 10^{-9} = 53 x 10^{-12} m

d) All the previous



How to convert same quantity with different prefixes in SI Units (2nd Case: units with power)

Same rule with powering the conversion factor with the same power of the given prefix or given unit.



Example 7:

✱ How many bottles of volume 10000 cm^3 is enough to fill a tank of capacity 1 m^3?

(a) 1 (b) 10 (c) 1000 (d) 100

Answer:

1st transfer m^3 to cm^3 to able to compare between two values.

Power

Given Number with **given** prefix ✖ ⟨ 1 Unit = ???? Of the **Required** Prefix **(R)** / 1 Unit = ???? Of the **Given** Prefix **(G)** ⟩ = New Number with **Required** prefix

$$1\ m^3 \times \left(\frac{100\ cm}{1\ m}\right)^3 = 1\ m^3 \times \frac{10^6\ cm^3}{1\ m^3} = 1 \times 10^6\ cm^3$$

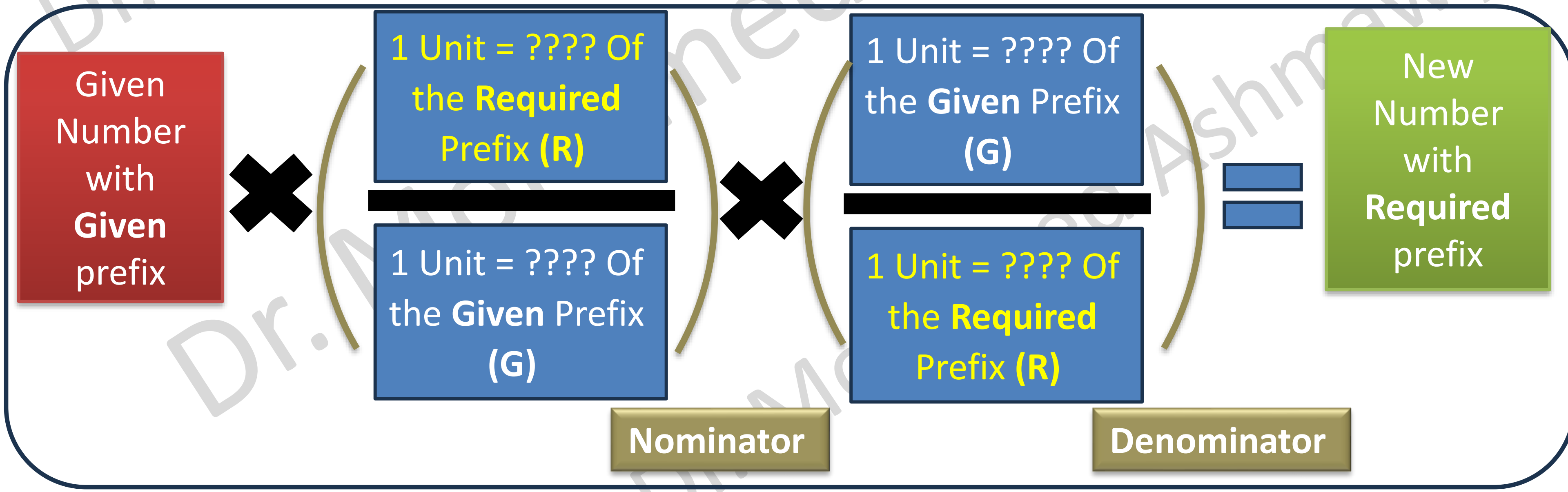
2nd then find number

$1 \times 10^6\ cm^3$ = number of bottles x $10^4\ cm^3$

So, number of bottles will $10^2 = 100$

How to convert same quantity with different prefixes in SI Units (3rd Case: nominator/denominator units convert)

Same previous rule for nominator and reciprocal for denominator



Example 8:

✱ If the speed of a car is 36 $km.h^{-1}$, then it is equivalent to ....

(a) 10 $m.s^{-1}$ (b) 20 $m.s^{-1}$ (c) 36 $m.s^{-1}$ (d) 100 $m.s^{-1}$

Answer:

Given Number with **Given** prefix ✖ (1 Unit = ???? Of the **Required** Prefix **(R)** / 1 Unit = ???? Of the **Given** Prefix **(G)**) ✖ (1 Unit = ???? Of the **Given** Prefix **(G)** / 1 Unit = ???? Of the **Required** Prefix **(R)**) = New Number with **Required** prefix

Nominator **Denominator**

$$36\,\frac{km}{h} = 36\,\frac{\cancel{km}}{\cancel{h}}\,x\,\left(\frac{1\ m}{10^{-3}\ \cancel{km}}\right)\,x\,\left(\frac{\frac{1}{3600}\ \cancel{h}}{1\ s}\right) = 36\,x\,\frac{10^{3}\ \ m}{3600\ \ s} = 10\,\frac{m}{s}$$

Example 9:

The density of iron is $7.86\ g/cm^3$ under standard conditions. Convert this to kg/m^3.

Answer:

Given Number with **Given** prefix ✖ (1 Unit = ???? Of the **Required** Prefix **(R)** / 1 Unit = ???? Of the **Given** Prefix **(G)**) ✖ (1 Unit = ???? Of the **Given** Prefix **(G)** / 1 Unit = ???? Of the **Required** Prefix **(R)**) = New Number with **Required** prefix

Nominator **Denominator**

$$7.86\,\frac{g}{cm^3} = 7.86\,\frac{g}{cm^3}\,x\left(\frac{10^{-3}kg}{1\ g}\right)x\left(\frac{10^{2}cm}{1\ m}\right)^3 = 7.86\,x\,\frac{10^6\ kg}{10^3 m^3} = 7.86x10^3\,\frac{kg}{m^3}$$

Significant Figures (Digits), it is rule that can express on the uncertainty in the measurement such that *the last digit written down in a measurement is the first digit with some uncertainty*.

This method is existed with tools (*OR given in the problem*) to help identifying the uncertainty instead of estimating it by eyes as in the following.

How to count significant FIGURES:

Ex: 503 3SF

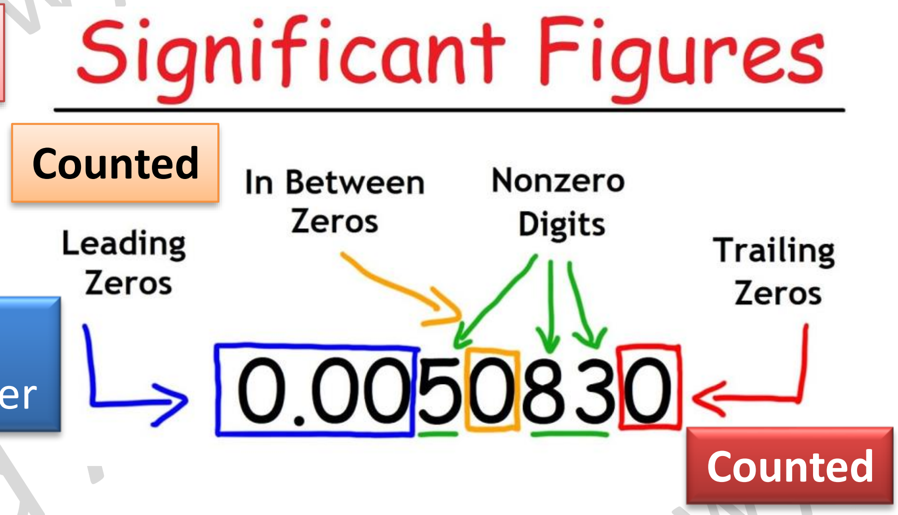
Ex: 503.0 4SF

Ex: 503.00 5SF

Ex: 00503 3SF

Ex: 00.0503 3SF

Not Counted
Called: Place Keeper



Example 6 (i):

Determine the number of significant figures in the following measurements

a. 0.0009

b. 15,450.0

c. 6×10^3

d. 87.990

e. 30.42

Solution

(a) one; the zeros in this number are placekeepers that indicate the decimal point

(b) six; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) one; the value 10^3 signifies the decimal place, not the number of measured values

(d) five; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) four; any zeros located in between significant figures in a number are also significant

Example 6 (j):

How many significant figures are in the following numbers?

a) 3.788×10^9 *4 SF*

b) 2.46×10^{-6} *3 SF*

c) 0.0053 *2 SF*

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*

$$\begin{array}{r} 7.56\ \text{kg} \\ -\ 6.052\ \text{kg} \\ +13.7\ \ \text{kg} \\ \hline 15.208\ \text{kg} \end{array} = 15.2\ \text{kg}.$$

$$A = \pi r^2 = (3.1415927\ldots) \times (1.2\ \text{m})^2 = 4.5238934\ \text{m}^2$$

$$A = 4.5\ \text{m}^2,$$

Example 6 (k):

Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of $0.0255\ \text{m/s}^2$, what is the force on the wagon? The unit of force is called the newton, and it is expressed with the symbol N.

Solution

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

3) Dimensional Formula

It is a form that represents **derived quantities *(infinite numbers)*** in terms of **fundamental quantities** (Length [L], Mass [M], and Time [T]); all must be in capital letters.

$$[A] = [M^{\pm a} L^{\pm b} T^{\pm c}]$$

Important Notes :

1) M not meter (m), it indicates to Mass
L not litter (L), it indicates to Length
T not Tension (T), it indicates to time

2) If the dimensions of both sides are ***identical, so this relation may be correct (not for sure), but*** if the dimensions are ***not the same, so the relation must be incorrect***

3) The dimensional formula ***could be multiplied or divided*** but ***couldn`t be added or subtracted unless*** they have the same unit, and the result will be the same without any numbers.

Do Not Forget

6 Rules of exponents

Rule	Example
$x^0 = 1$	$(2^0) = 1$
$x^1 = x$	$(-4)^1 = -4$
$x^{-m} = \frac{1}{x^m}$	$(3)^{-2} = \frac{1}{(3)^2} = \frac{1}{9}$
$(x^m)^n = x^{mn}$	$(2^2)^3 = (2)^{2\times 3} = (2)^6 = 64$
$(xy)^m = x^m y^m$	$(2\times 3)^2 = (2)^2 \times (3)^2 = 36$
$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{1}{3}\right)^2 = \frac{(1)^2}{(3)^2} = \frac{1}{9}$
$x^m x^n = x^{m+n}$	$(2)^3 \times (2)^{-2} = (2)^{3+(-2)} = (2)^1 = 2$
$\frac{x^m}{x^n} = x^{m-n}$	$\frac{(3)^4}{(3)^{-2}} = (3)^{4-(-2)} = (3)^6 = 729$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$	$(8)^{\frac{1}{3}} = \sqrt[3]{8} = 2$

Example 7:

What is the dimensional formula of velocity?

Solution:

∵ velocity [v]= distance / time = [L]/[T] = [L][T^{-1}] = [$M^0L^1T^{-1}$]

Example 8:

Verify the relation of the volume of a cube, Volume (V)= $(\text{Length})^3$.

Solution: ∵ L.H.S unit is known to be m^3, so its dimensional formula is [L^3]= [$M^0L^3T^0$]

∵ R.H.S is $(\text{Length})^3$ = m^3 and also its dimensional is [L^3].

$\therefore L.H.S. = R.H.S.$

So, the relation ***may be correct (possible)***.

Example 9:

Verify the relation of the volume of cylinder V= 2πr. h.

Solution:

∵ L.H.S unit is known to be m^3, so its dimensions is $[L^3]$

∵ R.H.S is 2πr. h $= [L].[L] = [L^2]$

∴ $L.H.S. \neq R.H.S.$ So, the relation ***is not correct at all***.

Example 10:

Which of the following equations are dimensionally correct?

a) $v_f = v_i + ax$

$$\frac{[L]}{[T]} = \frac{[L]}{[T]} + \frac{[L]}{[T]^2}[L] \rightarrow \frac{[L]}{[T]} = \frac{[L]}{[T]} + \frac{[L]^2}{[T]^2} \rightarrow dimentionally\ incorrect$$

b) $y = (2\,m)\cos(kx)$, where $k = 2\,m^{-1}$. (here m is $meters$)

$$[L] = [L]\cos\left(\frac{1}{[L]}[L]\right) \rightarrow [L] = [L] \rightarrow dimentionally\ correct$$

Example 11:

If the dimensional formula of quantity A is ML^2T^{-2} and the dimensional formula of quantity B is ML^2T^{-2}, then the quantity (2B – A) ....

ⓐ has a dimensional formula of ML^2T^{-2}

ⓑ has a dimensional formula of $M^2L^4T^{-4}$

ⓒ has a dimensional formula of $M^3L^6T^{-6}$

ⓓ isn't a physical quantity