Lecture 1

Differential equation

$y' = \frac{dy}{dx} = Dx = f'(x) = f^{(1)}(x)$

$f^{(2)}(x)$ & $f^2(x)$

F_x , F_{xx} , F_{xy}

First order differential equation

$\frac{d^2y}{dx^2} = y''$
second or.

$y' = \frac{dy}{dx}$ ← order
First order

degree

4 ← degree

$(y'')^2 + (y')^4 + y = 2$

Second order / Fourth degree

$(y')^6 + y^2 = 7$

order : First

degree : sixth

$\left(\frac{d^4y}{dx^4}\right)^2 + y^7 = 2$

order : 4

degree : 2

$f^{(1)}(x)$, $f^{(2)}(x)$, $f^{(3)}(x)$

$\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, $\frac{d^4y}{dx^4}$

f', f'', f''', $f^{(IV)}$

Dx , D^2x , D^3x

* Derivative

* integration

Method

step 1 →

step 2 → $\int$

$y = x^3 + 2x + 1$

$y' = 3x^2 + 2$

$y = \sin x$

$y' = \cos x$

$\int (x^2 + 2x + 1)\,dx$

$= \frac{x^3}{3} + \frac{2x^2}{2} + x$

$\int e^x\,dx = e^x$

$\int \frac{dx}{1+x} = \ln(1+x)$

Solve the differential equation

sep
homo
nonhom
Exact
Linear

Partial fraction

$$\int \frac{3}{x^2 + 5x + 6}\,dx$$

$$\int \frac{3}{(x+2)(x+3)} = \int \frac{A}{x+2} + \int \frac{B}{x+3}$$

$$= A \ln(x+2) + B \ln(x+3)$$

Ex

$$\int \frac{d^2y}{dt^2} = \int -g\,dt +$$

$f(t)$

$y(t)$

$$\frac{dy}{dt} = -gt + C$$

$$\int \frac{dy}{dt} = -g\int t\,dt + \int c\,dt$$

$\frac{dy}{dt}$ y'

$\frac{d^2y}{dt^2}$ y''

$\frac{d^3y}{dt^3}$ y'''

y

$$\boxed{y = -g\frac{t^2}{2} + ct + C_2}$$

initial condition $\{ \; y(0) = 1$

$y(1) = 2$

$$\boxed{1 = C_2}$$

$$2 = -g + c + 1$$

Ex $\; y = \underbrace{x^3 + 2x + 1}$

$$y' = 3x^2 + 2$$

$$\boxed{\begin{aligned} y &= x^3 + 2x + 3 \\ y' &= 3x^2 + 2 \end{aligned}}$$

$$\int (3x^2 + 2)\,dx$$

$$= 3\int x^2\,dx + 2\int dx$$

$y = x^3 + 2x$

$y' = 3x^2 + 2$

$$= \cancel{3}\frac{x^3}{\cancel{3}} + \frac{\cancel{2}x}{1} + C$$

$$\boxed{y = \underbrace{x^3 + 2x + C}}$$

1 Separable D E

فصل

Ex 1

$$\frac{dy}{dx} = \frac{2xy}{1}$$ solve the D E

Sol

$$dy = 2xy \, dx$$

divided by y

$$\int \frac{dy}{y} = \int 2x \, dx$$

$$\boxed{\ln(y) = \frac{2x^2}{2} + C}$$

$$\int \frac{f'(x)}{f(x)} \Longrightarrow \ln(f(x))$$

Ex 2 solve the D E

$$x \, dy = y \, dx$$

Sol

$$\frac{x}{y} dy = dx$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\boxed{\ln y = \ln x + C}$$

EX

$y' = e^{x+y}$ solve the DE

Sol

$\frac{dy}{dx} = e^{x+y}$

$\frac{dy}{dx} = \frac{e^x e^y}{1}$

$dy = e^x e^y \, dx$

$\int \frac{dy}{e^y} = \int e^x \, dx$

$-\int -e^{-y} dy$

$= -e^{-y}$

$\int 2x e^{x^2} dx = e^{x^2}$

$e^f = f' e^f$

$\int f' e^f dx = e^f$

$* \; e^{x+y} = e^x e^y$

$* \; e^{x-y} = \frac{e^x}{e^y}$

$x^2 \cdot x^3 = x^5$

$\frac{x^5}{x^2} = x^3$

$\int e^{-y} dy = \int e^x dx$

$\boxed{-e^{-y} = e^x + C}$

Ex $\dfrac{dy}{dx} = \dfrac{1}{x^2(1+y^2)}$

solve the DE

Sol

$$x^2(1+y^2)\,dy = dx$$

$$\int (1+y^2)\,dy = \int \frac{dx}{x^2}$$

$$\int dy + \int y^2 dy = \int x^{-2} dx$$

$$\boxed{y + \frac{y^3}{3} = -x^{-1} + c}$$

$\int dx = x$ $\int dz = z$

$\int x^0 dx = x$

EX $xy' + y = 0$

Solve the DE

Sol

$$x\frac{dy}{dx} = \frac{-y}{1}$$

$$x\,dy = -y\,dx$$

divided by xy

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\boxed{\ln y = -\ln x + c}$$

EX $y\,dx - x\,dy = xy\,dx$

Solve the DE

Sol

$$y\,dx - xy\,dx = x\,dy$$

$$(y - xy)dx = x\,dy$$

$$y(1-x)\,dx = x\,dy$$

divided by xy

$$\int \frac{(1-x)\,dx}{x} = \int \frac{dy}{y}$$

$$\int \frac{1}{x}dx - \int dx = \int \frac{dy}{y}$$

$$\ln x - x + c = y$$

Ex $y' = \frac{y^2+1}{t+1}$

Solve the DE

Sol

$$\frac{dy}{dt} = \frac{y^2+1}{t+1}$$

$$(t+1)\,dy = (y^2+1)\,dt$$

$(t+1)(y^2+1)$

$$\frac{dy}{y^2+1} = \frac{dt}{t+1}$$

$$\boxed{\tan^{-1} y = \ln(t+1) + C}$$

Ex $y - xy' = 3 - 2x^2y'$

Solve the DE

Sol

$$2x^2y' - xy' = 3 - y$$

$$(2x^2 - x)\frac{dy}{dx} = 3 - y$$

$$\frac{dy}{dx} = \frac{3-y}{2x^2-x}$$

$$(2x^2-x)\,dy = (3-y)\,dx$$

$$\int \frac{dy}{3-y} = \int \frac{dx}{2x^2-x}$$

EX Solve the DE

$$(x^3+x^2)y\,dx + x^2(y^3+2y)\,dy = 0$$

sol

divided by x^2y

$$\frac{x^3+x^2}{x^2}\,dx + \frac{(y^3+2y)}{y}\,dy = 0$$

$$\frac{(y^3+2y)}{y}\,dy = -\frac{x^3+x^2}{x^2}\,dx$$

$$\int (y^2+2)\,dy = -\int (x+1)\,dx$$

$$\boxed{\frac{y^3}{3}+2y = -\frac{x^2}{2} - x + C}$$

EX $(1+x^2)y' = (1+y^2)$

sol

$$(1+x^2)\frac{dy}{dx} = \frac{(1+y^2)}{1}$$

$$(1+x^2)\,dy = (1+y^2)\,dx$$

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\boxed{\tan^{-1} y = \tan^{-1} x + C}$$