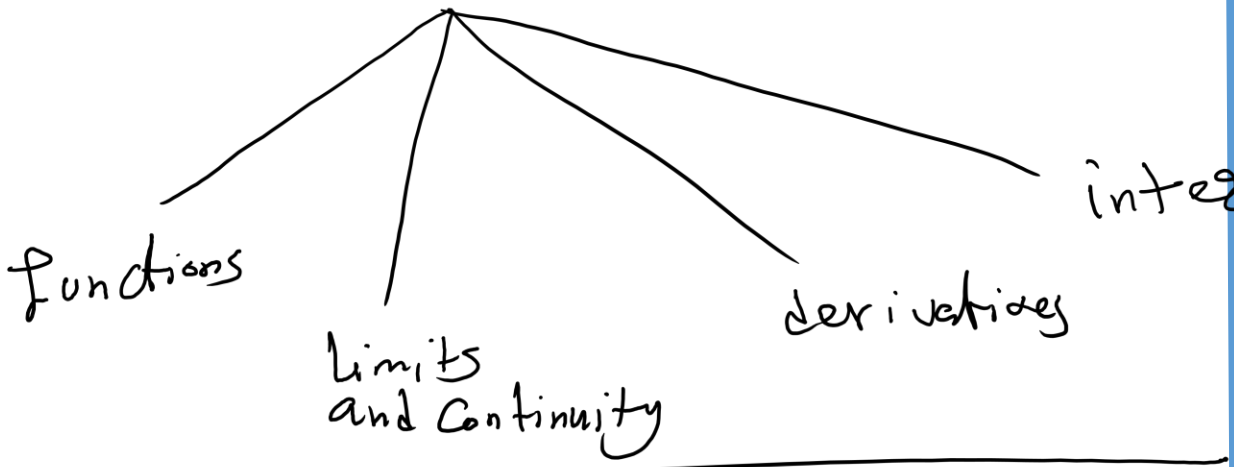
Lecture 1

Calculus I



Lines, circles

* absolute value

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$|-2| = 2$

$|2| = 2$

[1] $\frac{|3|}{x} = \frac{3}{x}$

[2] $\frac{|-5|}{x} = \frac{5}{x}$

[*] $|-a| = |a|$

$|-5| = |5| = 5$

[*] $|ab| = |a|\,|b|$

$|5a| = |5|\,|a|$

[*] $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

[*] $|a+b| \leq |a| + |b|$ triangle inequality

$|-5+2| \leq |-5| + |2|$

$|-3| = 3 \leq 5 + 2 = 7$

$|x| = \sqrt{x^2}$

$|5| = \sqrt{5^2}$

$= \sqrt{25} = 5$

$\sqrt{a^2} = |a|$

Equation

1) $|x+2| = 3$

$(x+2) = 3 \Rightarrow x = 3-2$

$\boxed{x = 1}$

$-(x+2) = 3$

$-x - 2 = 3$

$-x = 3+2$

$-x = 5$

$\boxed{x = -5}$

$|x-1| = 2$

$x-1 = 2$ $\quad \boxed{x = 3}$

$-(x-1) = 2$

$-x+1 = 2$

$-x = 2-1$

$-x = 1$

$\boxed{x = -1}$

* $|x+2| \leq 3$

$\leq$ $\geq$ $<$ $>$

interval الفترات

[1] closed interval

$[a, b]$

$y = x+1$

$[3, 9]$

[2] open interval

(a, b)

$(3, 9)$

3 x

↳ 3.000001

x 9

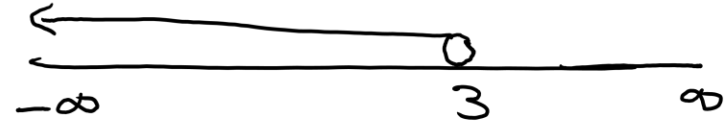
↳ 8.999999

[3] half open

$(a, b]$

$[a, b)$

* $x < 3$



$(-\infty, 3)$

* $x \leq 3$

$(-\infty, 3]$

* $x > 3$

$(3, \infty)$

* $3 < x \leq 7$

$(3, 7]$

$2 \leq x \leq 5$

$[2, 5]$

$0 < x < 5$

$(0, 5)$

* $|x+2| \leq 3$

$-3 \leq x+2 \leq 3$

$-3-2 \leq x+2-2 \leq 3-2$

$-5 \leq x \leq 1$

$[-5, 1]$

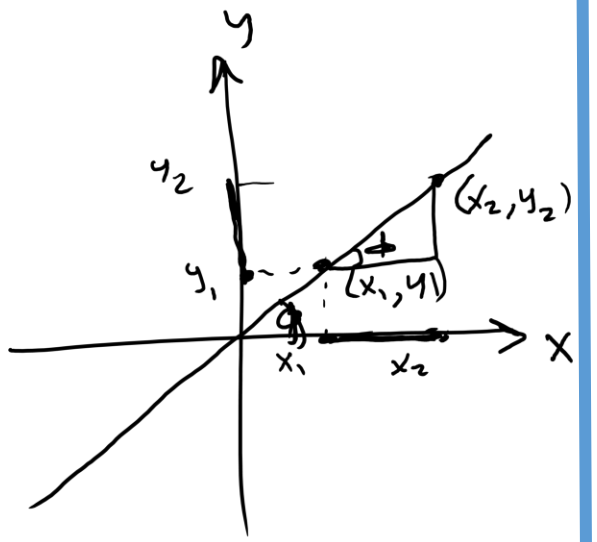
* $|x+2| > 3$

Lines

* $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

slope

(x_1, y_1) (x_2, y_2)



$$\frac{m}{1} = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y = y_1 + m(x - x_1)}$$ ← equation for the line through the point

equation of tangent line

EX write an equation for the line through the point $(2, 3)$ with slope $-3/2$

$= (x_1, y_1) = (2, 3)$

$m = -3/2$

Sol

$$y = y_1 + m(x - x_1)$$

$$y = 3 + \frac{-3}{2}(x - 2)$$

$$y = 3 - \frac{3}{2}x + 3$$

$$\boxed{y = -\frac{3}{2}x + 6}$$ ← y-intercept

$$y = mx + b$$

↑ slope

* $(x_1, y_1) = (1, 2)$ $\quad m = 3$

$$y = y_1 + m(x - x_1)$$

$$y = 2 + 3(x - 1)$$

$$y = 2 + 3x - 3$$

$$\boxed{y = 3x - 1}$$

Point , Slope

two points

Ex write an equation for the line through $(-2, -1)$ and $(3, 4)$

$x_2 \; x_2 \quad x_1 \; y_1$

So

1

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{3 - (-2)} = \frac{4 + 1}{3 + 2} = \frac{5}{5} = 1$$

$\boxed{m = 1}$

$$m = \frac{-1 - 4}{-2 - 3} = \frac{+5}{+5} = 1$$

$\boxed{m = 1}$ slope

2

$$y = y_1 + m(x - x_1)$$

$$y = -1 + (x + 2) = x + 1$$

$\boxed{y = x + 1}$

$$y = 4 + (x - 3) = x + 1$$

$\boxed{y = x + 1}$

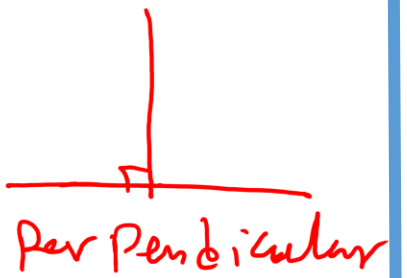
Parallel and Perpendicular lines

$L_1 \rightarrow y = x + 3$ ← $m = 1$

$L_2 \rightarrow y = -x + 1$ ← $m = -1$

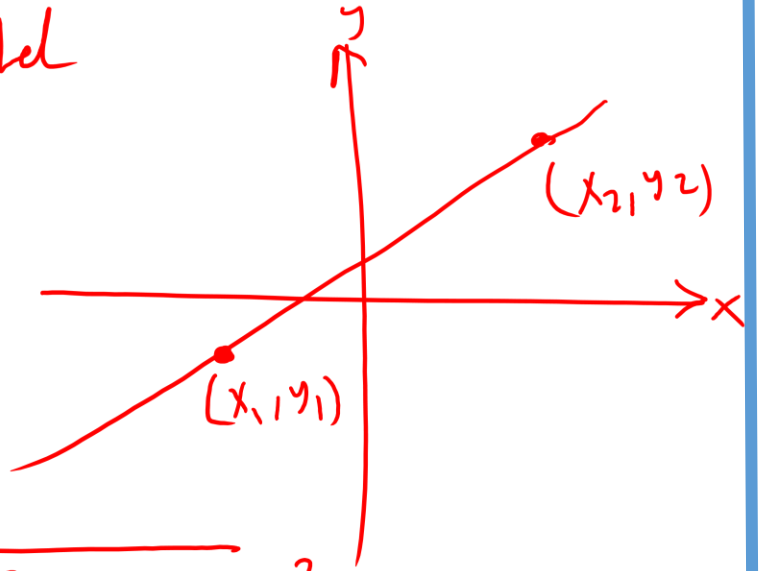
// Parallel

$m_1 m_2 = -1$ Perpendicular



$m_1 = m_2$ Parallel

distance between two points



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

□ ⟶ ( ) ( ) m_1

□ ⟶ ( ) ( ) m_2

Ex Find the distance between

$P(-1, 2)$, $Q(3, 4)$

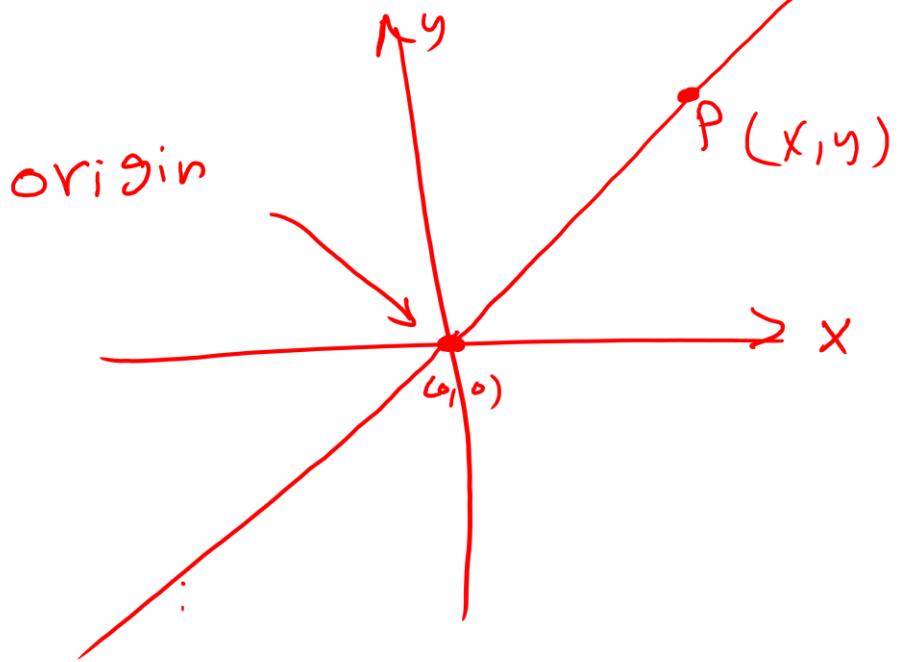
$x_1\ y_1$ $x_2\ y_2$

$$d = \sqrt{(3-(-1))^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2}$$

$$= \sqrt{16+4} = \sqrt{20}$$

$$= \sqrt{4*5} = 2\sqrt{5}$$

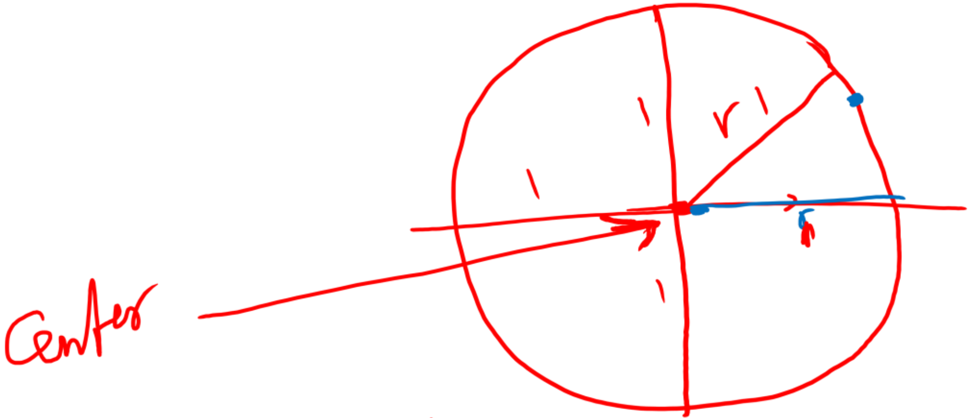


$$d = \sqrt{(y-0)^2 + (x-0)^2} = \sqrt{y^2 + x^2}$$

Ex Find the distance between the point (3, 7) and origin

$$d = \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

circle



unit circle

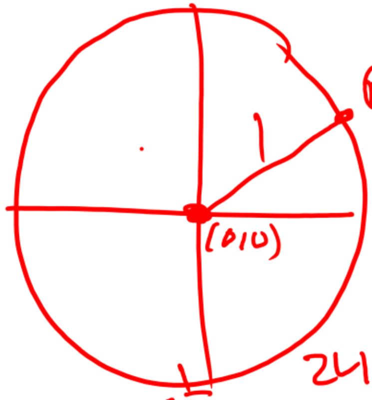
$r=1$

$$x^2+y^2=1$$

$$d=\sqrt{x^2+y^2}$$

$$1=\sqrt{x^2+y^2}$$

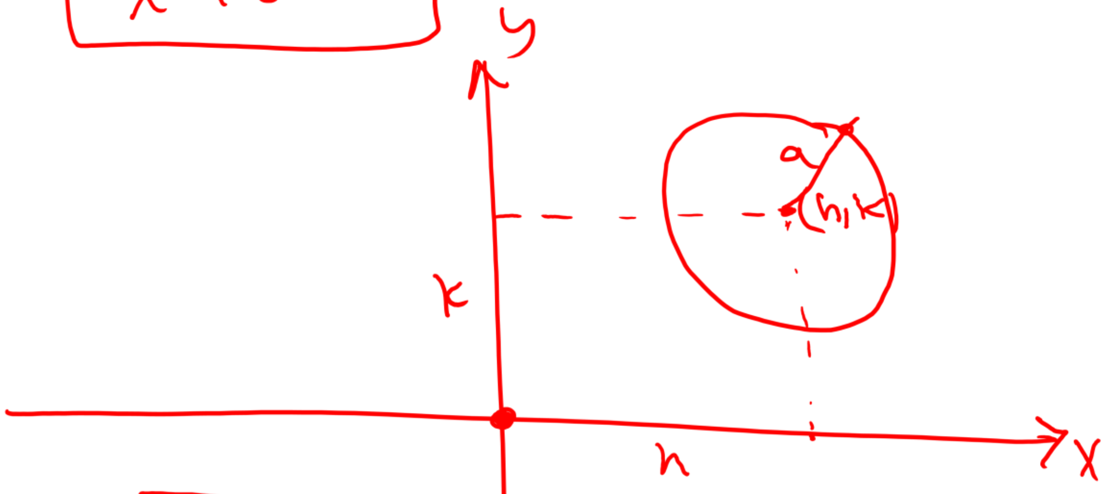
$$x^2+y^2=1$$



Cos
tan
cot sec
sin 30 45 90
120
150 = 1/2
210 = -1/2
240

unit circle: Center (0,0), $r=1$

$$x^2+y^2=1$$



$$d=\sqrt{(y-y_1)^2+(x-x_1)^2}$$

P(x,y) $r=\sqrt{(x-h)^2+(y-k)^2}$

Center (h,k)

$$(x-h)^2+(y-k)^2=r^2$$

a^2

Standard equation of circle