Mid-Term Revision

Physics1

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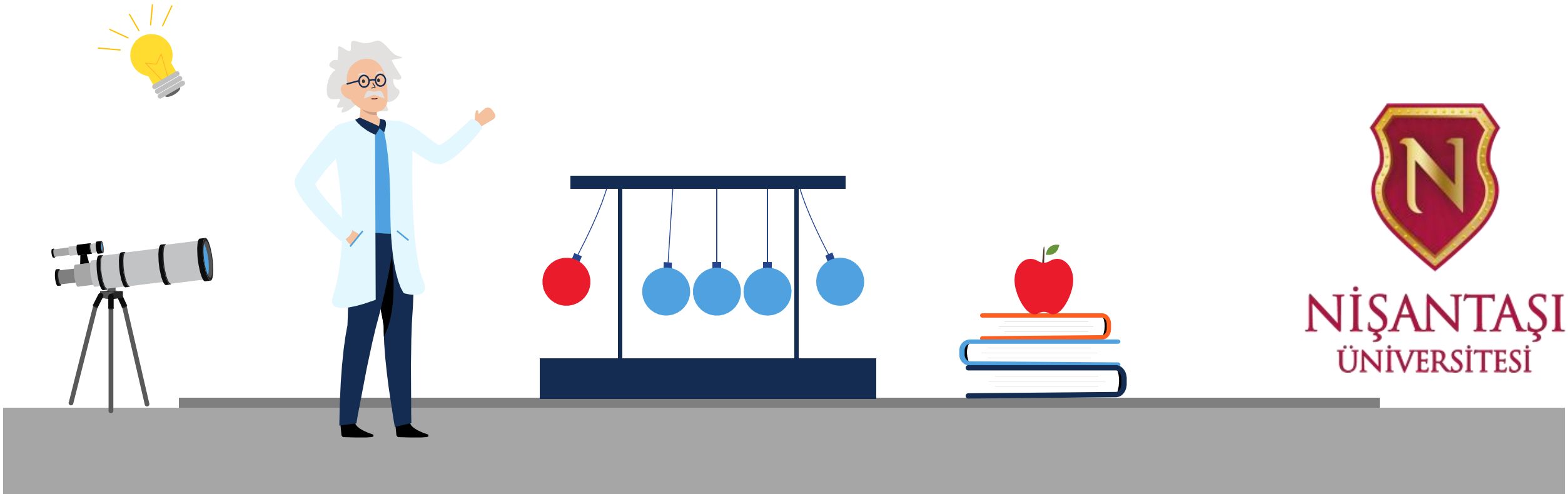
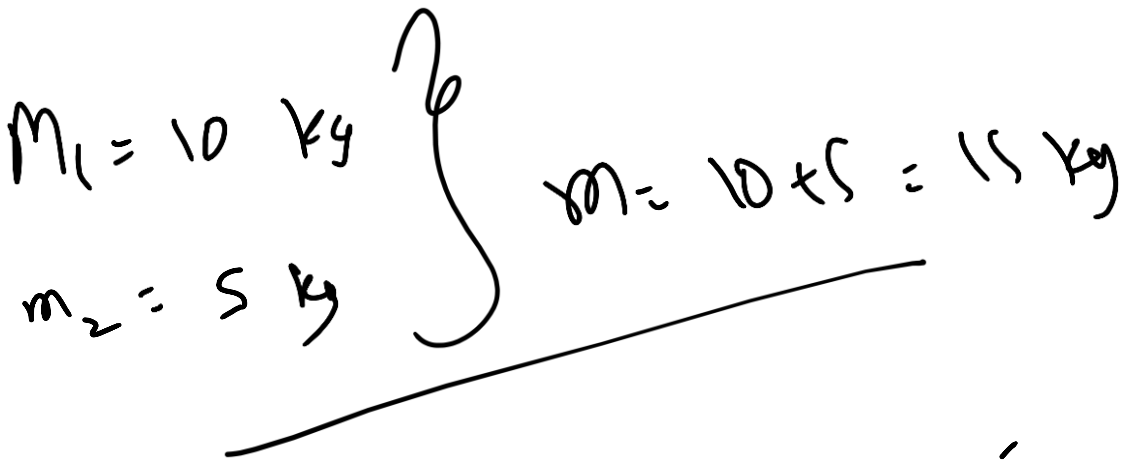
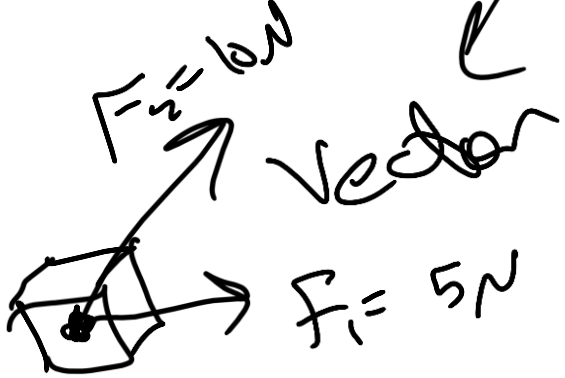


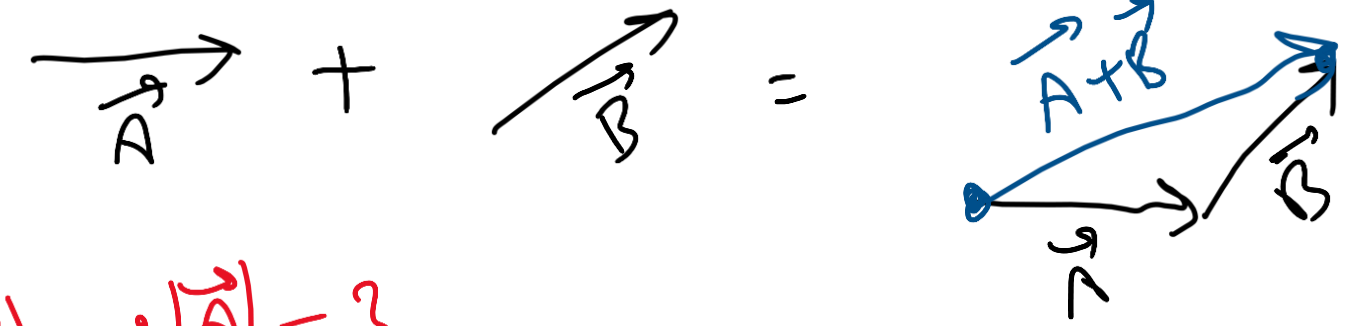
Vectors and scalars

- A **scalar quantity** can be described by a *single number*.
- A **vector quantity** has both a *magnitude* and a *direction* in space.
- In this book, a vector quantity is represented in boldface italic type with an arrow over it: $\vec{A}$.
- The magnitude of $\vec{A}$ is written as A or $|\vec{A}|$.



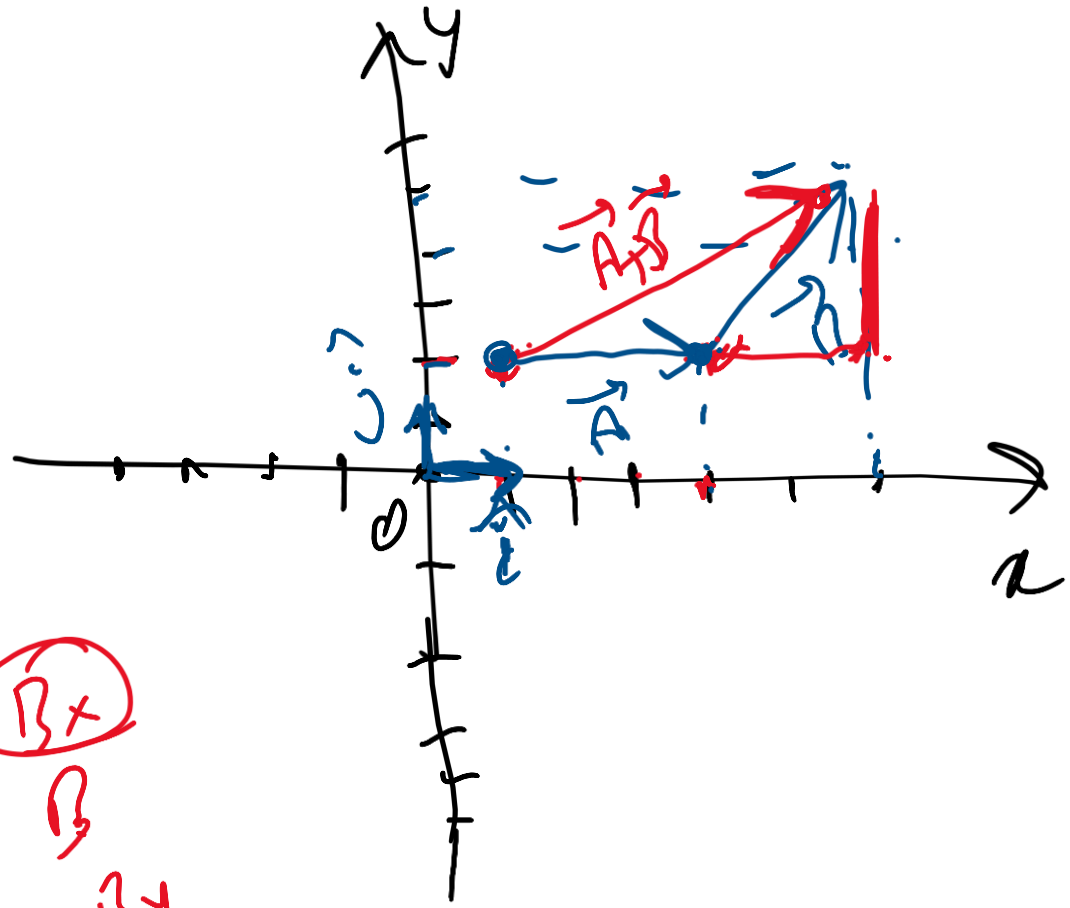


1. $\vec{A} + \vec{B}$



$\vec{A}$: $(1,2) \rightarrow (4,2)$, $|\vec{A}| = 3$

$\vec{B}$: $(4,2) \rightarrow (6,1)$



$B = \sqrt{B_x^2 + B_y^2}$

$\cos\theta = \frac{B_x}{B}$

$\tan\theta = \frac{B_y}{B_x}$

$\sin\theta = \frac{B_y}{B}$

$A + B = R$

$A_x = 3$, $A_y = 0$

$B_x = 2$, $B_y = 3$

$R_x = A_x + B_x$

$R_y = A_y + B_y$

$R_x = 3 + 2 = 5$

$R_y = 0 + 3 = 3$

$R = \sqrt{(5)^2 + (3)^2} =$

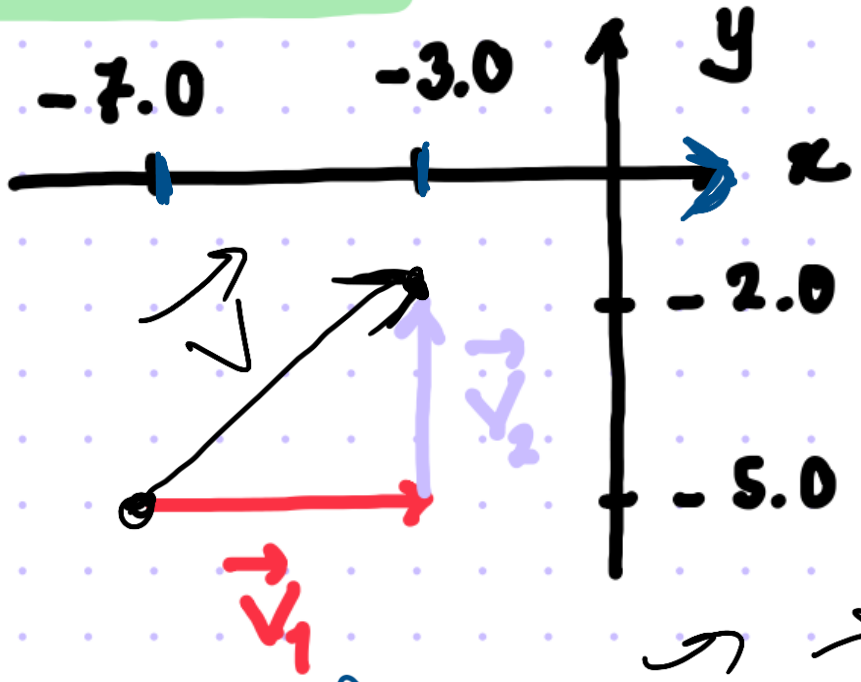
Unit Vector notation

$\vec{A}$: $A_x = 3$, $A_y = 0$ $\Rightarrow$ $\vec{A} = 3\hat{i}$ $(+0\hat{j})$

$\vec{B}$: $B_x = 2$, $B_y = 3$ $\Rightarrow$ $\vec{B} = 2\hat{i} + 3\hat{j}$

$$\vec{R} = 5\hat{i} + 3\hat{j} \rightarrow |\vec{R}| = \sqrt{5^2 + 3^2} =$$

Question 1.



Please find

i. The magnitude of the resultant vector, and

ii. The direction of the resultant vector.

iii. Sketch the resultant vector.

$V_1 = 4\hat{i}$

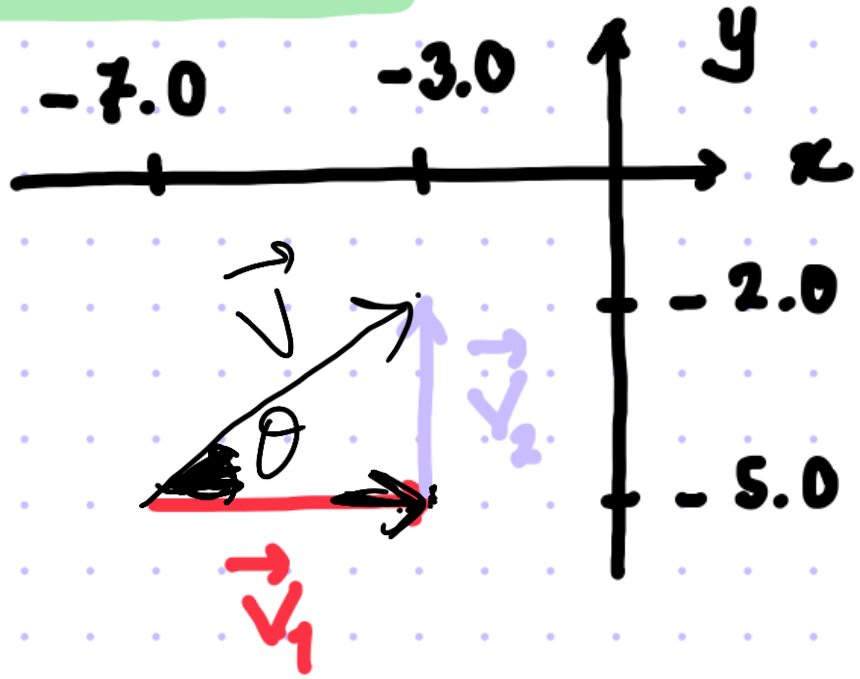
$V_2 = 3\hat{j}$

$\vec{V} = \vec{V_1} + \vec{V_2}$

$\vec{V} = 4\hat{i} + 3\hat{j}$

$|\vec{V}| = \sqrt{4^2 + 3^2} = 5$

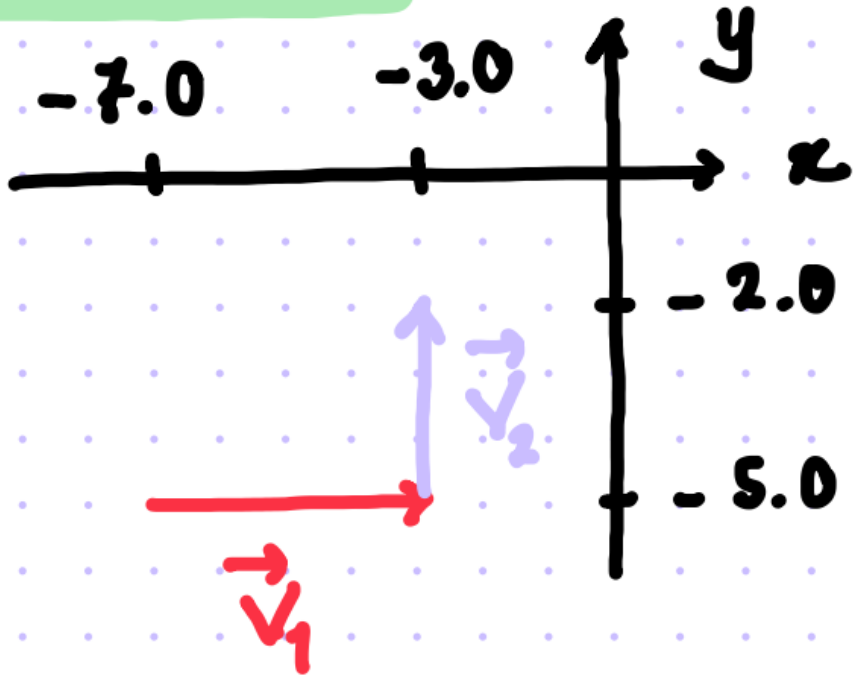
Question 1.



(ii) θ $\quad \vec{V} = 4\hat{i} + 3\hat{j}$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

Question 1.



Question 2.

Please consider the two vectors as follows:

$\vec{A} = 3.0\hat{i} + 4.0\hat{j}$ and $\vec{B} = -3.0\hat{i} - 5.0\hat{j} + 2.0\hat{k}$.

they are lying in the xyz-coordinate system.

Please find:

i. the resultant vector. ✓

ii. the magnitude of the resultant vector. ✓

iii. $3\vec{A} - \vec{B}$.

iv. $\vec{A} \cdot \vec{B}$.

v. $\vec{B} \cdot \vec{A}$.

vi. $\vec{A} \times \vec{B}$.

vii. $\vec{B} \times \vec{A}$.

$\vec{A} = 3.0\hat{i} + 4.0\hat{j}$ and $\vec{B} = -3.0\hat{i} - 5.0\hat{j} + 2.0\hat{k}$.

(i)

$$\vec{R} = \vec{A} + \vec{B} = (3.0 + (-3.0))\hat{i} + (4 + (-5))\hat{j} + (0 + 2)\hat{k}$$

$$\boxed{\vec{R} = -\hat{j} + 2\hat{k}}$$

(ii)

$$|\vec{R}| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

(iii)

$$3\vec{A} - \vec{B} = 3(3\hat{i} + 4\hat{j}) - (-3\hat{i} - 5\hat{j} + 2\hat{k})$$

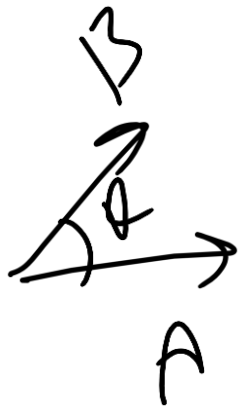
$$= (9\hat{i} + 12\hat{j}) - (-3\hat{i} - 5\hat{j} + 2\hat{k})$$

$$3\vec{A} - \vec{B} = 12\hat{i} + 17\hat{j} - 2\hat{k}$$

$\vec{A} \cdot \vec{B}$

↓

Scalar



$\vec{A} \times \vec{B}$

ضرب المتجهات

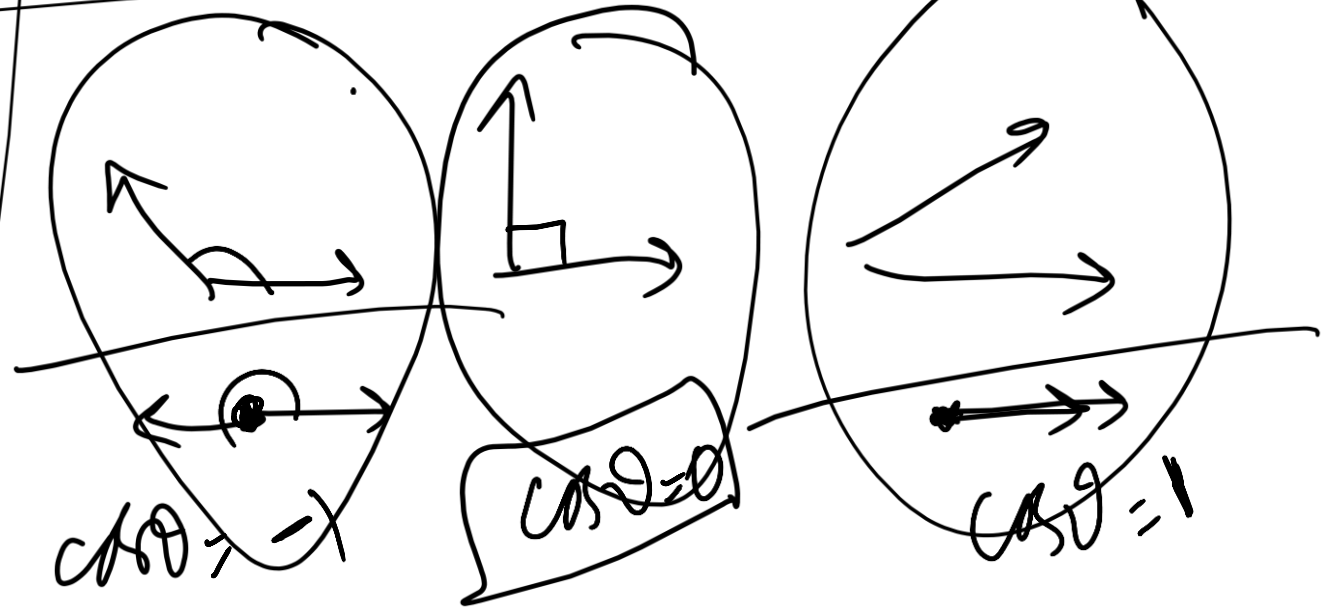
↓

Vector $\vec{C}$

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\theta$

$\vec{B} \cdot \vec{A} = |\vec{B}| \, |\vec{A}| \cdot \cos\theta$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$



$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j}$$

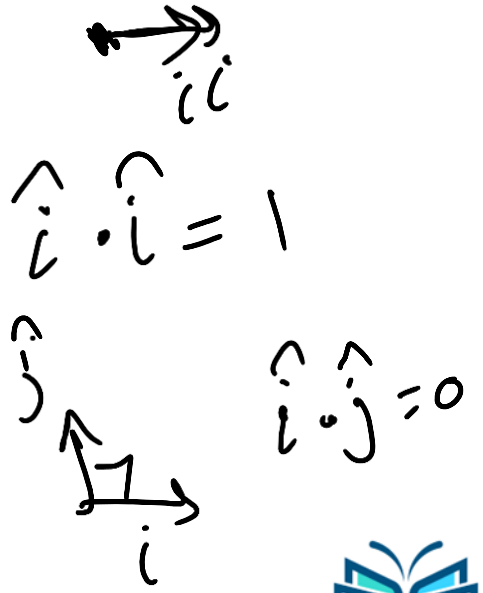
$\vec{A} = 3.0\hat{i} + 4.0\hat{j}$ and $\vec{B} = -3.0\hat{i} - 5.0\hat{j} + 2.0\hat{k}$.

$$\vec{A} \cdot \vec{B} = (3)(-3) + (4)(-5) + (0)(2)$$

$$= -9 - 20 + 0$$

$$= -29$$

$$\vec{B} \cdot \vec{A} = \vec{A} \cdot \vec{B} = -29$$

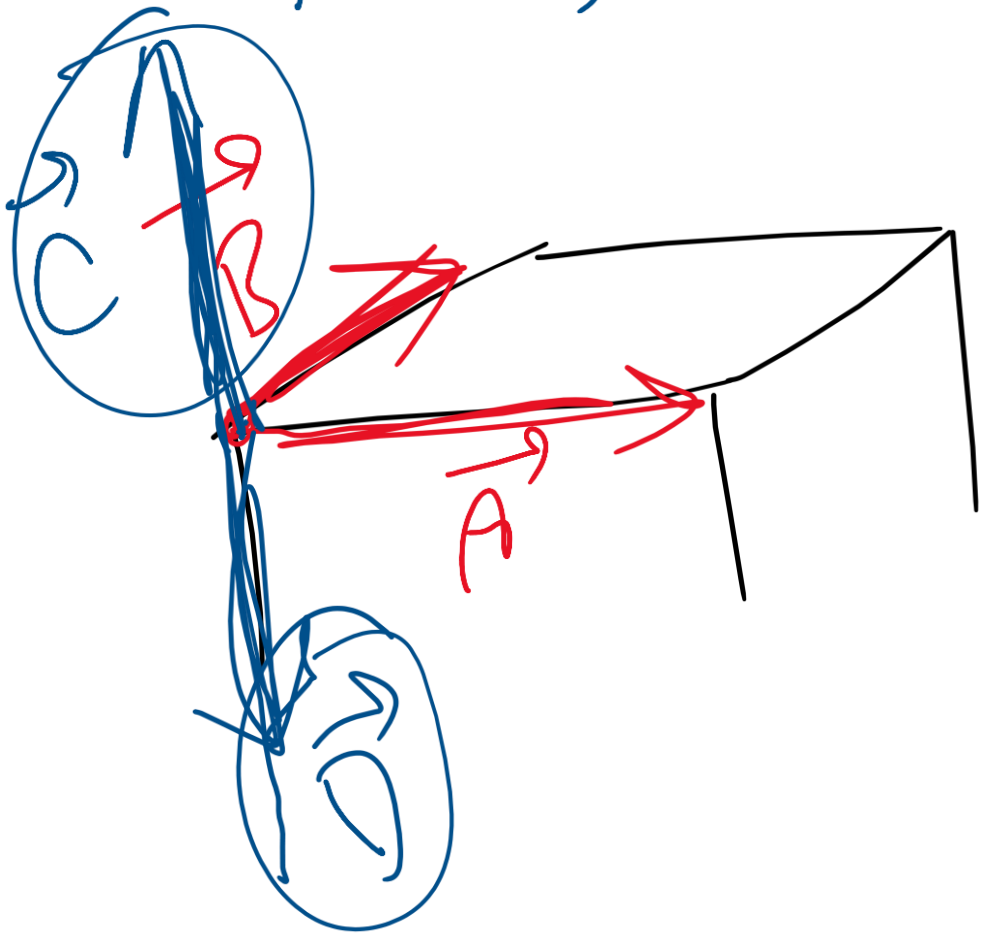


$\vec{A} \times \vec{B} = \vec{C}$ ← عكسية

$\vec{B} \times \vec{A} = \vec{D}$ ← موجبة

$|\vec{C}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta$

$\vec{C} \perp \vec{A}, \vec{B}$ ← حسب قاعدة اليمنى

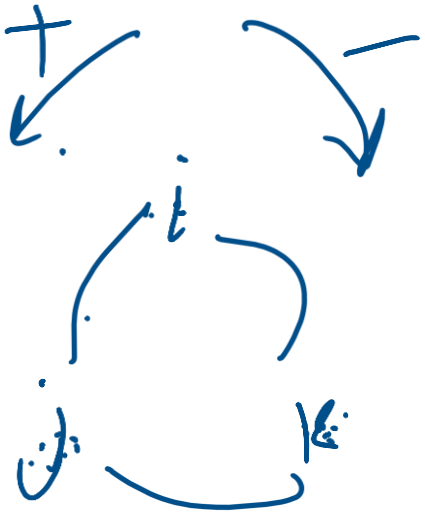


$\vec{D} = -\vec{C}$

$|\vec{D}| = |\vec{C}|$

$\vec{A} = 3.0\hat{i} + 4.0\hat{j}$ and $\vec{B} = -3.0\hat{i} - 5.0\hat{j} + 2.0\hat{k}$.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & -5 & 2 \end{vmatrix}$$



$\hat{i}\times\hat{i} = 0$

$\hat{j}\times\hat{j} = 0$

$\hat{k}\times\hat{k} = 0$

$= +\hat{i}\left[(4)(2) - (0)(-5)\right]$

$-\hat{j}\left[(3)(2) - (0)(-3)\right]$

$+\hat{k}\left[(3)(-5) - (4)(-3)\right]$

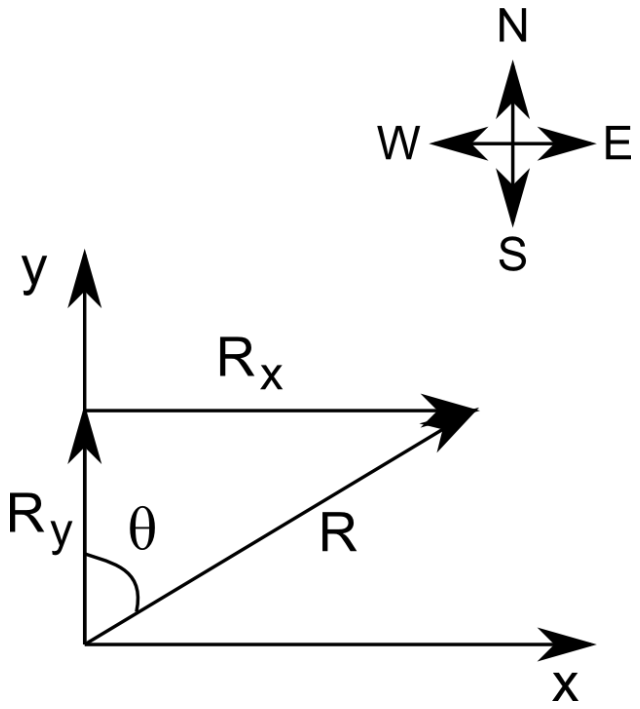
$= 8\hat{i} - 6\hat{j} - 3\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{8^2 + (-6)^2 + (-3)^2}$

$=$ ☐

Question 3:

If you will walk 1.00 km north and then 2.00 km east on a horizontal field, how far and in what direction is you from the starting point?



a) We can find the resultant displacement of the cross-country skier from the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.0km)^2 + (1.0km)^2} = 2.24km.$$

b) We can find the direction of the displacement vector from the geometry of the problem:

$$tan\theta = \frac{R_x}{R_y},$$

$$\theta = tan^{-1}(\frac{R_x}{R_y}) = tan^{-1}(\frac{2.0km}{1.0km}) = 63.4°.$$

Question 4:

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in the Figure. Find the magnitude and direction of the car's resultant displacement.

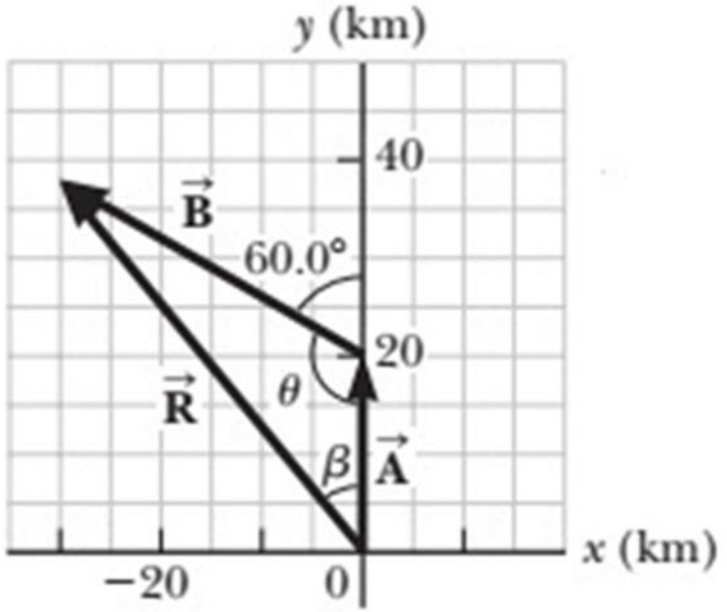
Use $R^2 = A^2 + B^2 - 2AB\cos\theta$ from the law of cosines to find R:

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Substitute numerical values, noting that $\theta = 180° - 60° = 120°$:

$$R = \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$

$$= 48.2\text{ km}$$



$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = 38.9°$$

Question 5:

Find the sum of two vectors $\vec{A}$ and $\vec{B}$ lying in the xy-plane and given by $\vec{A} = (2.0\,\hat{i} + 2.0\,\hat{j})$ and $\vec{B} = 2.0\,\hat{i} - 6.0\,\hat{j}$.

Question 6:

Given the two displacements

$\vec{D} = (6.00\hat{i} + 3.00\hat{j} - 1.00\hat{k})$ m and

$\vec{E} = (4.00\hat{i} - 5.00\hat{j} + 8.00\hat{k})$ m.

Find the magnitude of the displacement $2\vec{D} - \vec{E}$.

Question 7:

A particle undergoes three consecutive displacement:

$\Delta\vec{r}_1 = (15\hat{i} + 30\hat{j} + 12\hat{k})$ cm,

$\Delta\vec{r}_2 = (23\hat{i} - 14\hat{j} - 5.0\hat{k})$ cm, and

$\Delta\vec{r}_3 = (-13\hat{i} + 15\hat{j})$ cm.

Find unit-vector notation for the resultant displacement and its magnitude in terms of m.