Arithmetic operations:

1. $a(b+c) = ab + ac$

2. $\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$

3. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

4. $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c}$

$\frac{2}{3} + \frac{1}{3} = \frac{2+1}{3}$

$= \frac{3}{3} = \frac{2}{3} + \frac{1}{3}$

$\frac{1 \times 2}{3 \times 2} + \frac{2}{6}$

$\frac{2}{6} + \frac{2}{6}$

$\frac{4}{6}$

$\frac{a}{b} \div \frac{c}{d}$ KFC

$\frac{a}{b} \times \frac{d}{c}$

- Simplify:

(*) $3(x+6) + 3(2x-3)$

$= 3x + 18 + 6x - 9$

$= 9x + 9 = 9(x+1)$

$\Rightarrow 3(x+6+2x-3) = 3(3x+3)$

$3 \cdot 3(x+1) = 9(x+1)$

(*) $(x-1)^2 + (x-1)(x+5)$

$x^2 - 2x + 1 + x^2 - x + 5x - 5$

$2x^2 + 2x - 4 = 2(x^2 + x - 2)$

$2(x-1)(x+2)$

$(x-1)^2 = (x-1)(x-1)$

$(x-1)^2 = x^2 - 2x + 1$

Another sol:

$(x-1)[(x-1) + (x+5)]$

$(x-1)[2x+4] = 2(x-1)(x+2)$

(*) $\dfrac{(x-3)^2 + (6x-18)}{(x-3)}$

method 1

$\dfrac{x^2-6x+9+6x-18}{(x-3)} = \dfrac{x^2-9}{(x-3)} = \dfrac{(x-3)(x+3)}{(x-3)}$

x^2-9 → Diff. of 2 squares → $(x-3)(x+3)$

$= x+3$

method 2

$\dfrac{(x-3)^2+6(x-3)}{(x-3)} = \dfrac{(x-3)(x-3+6)}{(x-3)} = (x+3)$

Simplify: $\dfrac{x}{2y} + \dfrac{x+1}{2y} + \dfrac{2x+1}{2y}$

$\dfrac{x+x+1+2x+1}{2y} = \dfrac{4x+2}{2y} = \dfrac{2(2x+1)}{2y} = \dfrac{2x+1}{y}$

$\dfrac{2x}{y} + \dfrac{1}{y}$

(*) $\dfrac{2}{x+3} + \dfrac{4}{x} = \dfrac{2x+4(x+3)}{(x+3)(x)}$

$\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+bc}{bd}$

$\dfrac{2x+4x+12}{x^2+3x} = \dfrac{6x+12}{x(x+3)} = \dfrac{6(x+2)}{x(x+3)}$

(*) $\dfrac{\frac{x}{y^2}}{\frac{y}{x^2}} = \dfrac{x}{y^2} \times \dfrac{x^2}{y} = \dfrac{x^3}{y^3} = \left(\dfrac{x}{y}\right)^3$

Factorization:

(*) $x^3 - y^3 = (x - y)(x^2 + xy + x^2)$

factorize.

① $8t^3 - 27 \rightarrow x^3 - y^3$ $\quad 8 = 2^3$, $27 = 3^3$

$= (2t - 3)(4t^2 + 6t + 9)$

② $16y^3 - 250$

$2(8y^3 - 125) = 2(2y - 5)(4y^2 + 10y + 25)$

③ $x^6 - 64$

method 1: $(x^3)^2 - 8^2 = (x^3 - 8)(x^3 + 8)$

$(x-2)(x^2 + 2x + 4)(x+2)(x^2 - 2x + 4)$

$(x-2)(x+2)(x^4 - 2x^3 + 4x^2 + 2x^3 - 4x^2 + 8x + 4x^2 - 8x + 16)$

$(x-2)(x+2)(x^4 + 4x^2 + 16)$

$(x^2)^3 - 4^3$

$(x^2 - 4)(x^4 + 4x^2 + 16)$

$(x-2)(x+2)(x^4 + 4x^2 + 16)$

Binomial Theorem:

$(x+y)^{2} = x^2 + 2xy + y^2$

$(x+y)^{3} = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

* $(2x - 3y)^3 = 8x^3 - 3\cdot 4x^2 3y + 3\cdot 2x \cdot 9y^2 - 27y^3$

$8x^3 - 36x^2y + 54xy^2 - 27y^3$

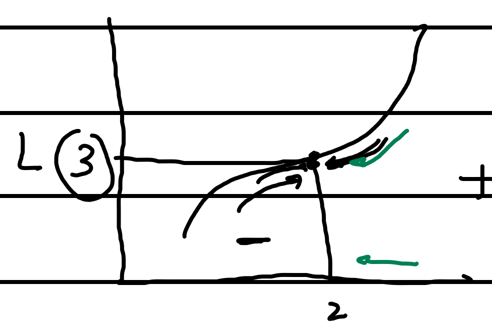
* $(2t^2 + z^3)^{3} = 8t^6 + 3\cdot 4t^4 z^3 + 3\cdot 2t^2z^6 + z^9$

$8t^6 + 12t^4z^3 + 6t^2z^6 + z^9$

$f(2)$

① Direct substitution-

$f(a) = L$



② $f(x) = \sqrt[n]{-ve}$ even

DNE

③ $f(a) = \frac{c}{0}$

$\lim_{a^+} = \infty$ / $-\infty$

$\lim_{a^-} = \infty$ / $-\infty$

$L = \infty$

$L = -\infty$

DNE

(4) $f(a) = \frac{0}{0}, 0^0, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 1^\infty, \infty^0, 1^\infty$

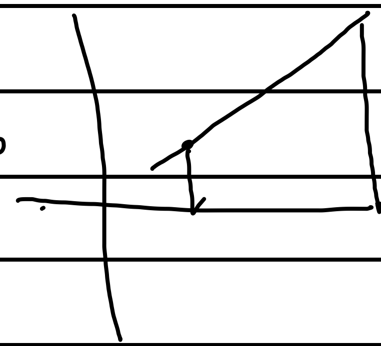
- → factorization
- → common denominator
- → conjugate multiplication
- → Expanding.
- → Absolute value
- → special forms : $\frac{\sin x}{x}$ - - .

Domain: values of x

$\frac{c}{0} \neq$ Denominator $\neq 0$

$\sqrt{\square} \geq 0$

$\sqrt{-ve}$ error.

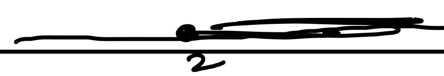


(*) $\frac{1}{x+1} \Rightarrow x+1 \neq 0$

$x \neq -1$ $x \neq -1$

$(-\infty, -1) \cup (-1, \infty) \Rightarrow R / \{-1\}$

* $\sqrt{x-2}$ $x - 2 \geq 0$ $x \geq 2$ $[2, \infty)$



* $\frac{1}{\sqrt{x-5}}$ $x - 5 > 0$

$x > 5$ $(5, \infty)$

① $\lim_{x \to 5} (2x^2 - 3x + 4)$

$D = R$ $(-\infty, \infty)$ any polynomial function has a limit

$f(5) = 2(5)^2 - 3(5) + 4 =$

$2(25) - 15 + 4 = 39 \to L$

$x - 1$

$x^2 + 2x$

$x^3 + 2$

② $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

$5 - 3x \neq 0$

$-3x \neq -5$

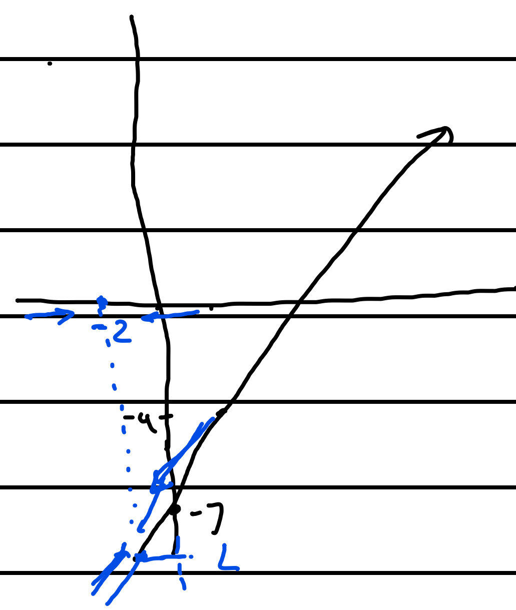
$x \neq \frac{5}{3}$

$f(-2) = \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = \frac{-8 + 8 - 1}{11} = \frac{-1}{11} \to L$

③ $\lim_{x \to -2} (3x - 7)$

x	0	1
f(x)	-7	-4

$f(-2) = 3(-2) - 7 = -6 - 7 = -13$



④ $\lim_{x \to 6} (8 - \frac{1}{2}x)$

$f(6) = 8 - \frac{1}{2}(6) = 5$

⑤ $\lim_{x \to 2} \frac{x^2 + 5x + 4}{x - 2}$

$x - 2 \neq 0$

$x \neq 2$

$f(2) = \frac{2^2 + 5(2) + 4}{2 - 2} = \frac{18}{0} = \pm\infty$

$\lim_{x \to 2^-} \frac{18}{-} = -\infty$

limit DNE

$\lim_{x \to 2^+} \frac{18}{+} = +\infty$

$f(x) = g(x)$

$\frac{x^2-1}{(x-1)} = x+1$

$= 2$

(6) $\lim_{x\to 1} \frac{x^2-1}{x-1}$ $f(x)$

$f(1) = \frac{1-1}{1-1} = \frac{0}{0}$

$\lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = x+1 = 2 \to L$ $g(x)$

(7) $\lim_{x\to 4} \frac{t^2 - 2t - 8}{t-4}$

8: 2 4

$f(4) = \frac{4^2 - 2(4) - 8}{4-4} = \frac{16-8-8}{0} = \frac{0}{0}$

$\lim_{x\to 4} \frac{(t-4)(t+2)}{t-4} = \lim_{x\to 4} (t+2) = 6 \to L$

(8) $\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} =$

$f(-3) = \frac{(-3)^2 + 3(-3)}{(-3)^2 - (-3) - 12} = \frac{0}{0}$

$\lim_{x\to -3} \frac{x(x+3)}{(x-4)(x+3)} = \lim_{x\to -3} \frac{x}{x-4} = \frac{-3}{-3-4} = \frac{-3}{-7} = \frac{3}{7}$

(9) $\lim_{x\to 4} \frac{x^2+3x}{x^2-x-12}$

$f(4) = \frac{4^2 + 3(4)}{4^2 - 4 - 12} = \frac{28}{0}$

$\lim_{x\to 4} \frac{x(x+3)}{(x-4)(x+3)} = \lim_{x\to 4} \frac{x}{(x-4)} = \frac{4}{\infty}$

$\lim_{x\to 4^-} \frac{4}{4^- - 4} = -\infty$ $\lim_{x\to 4^+} \frac{4}{4^+ - 4} = \infty$

limit DNE

(10) $\lim_{x \to -2} \frac{x^2 - x - 6}{3x^2 + 5x - 2} =$

$f(-2) = \frac{(-2)^2 - (-2) - 6}{3(-2)^2 + 5(-2) - 2} = \frac{4 + 2 - 6}{12 - 10 - 2} = \frac{0}{0}$

$\lim_{x \to -2} \frac{x^2 - x - 6}{3x^2 + 5x - 2} = \frac{(x-3)(x+2)}{(x+2)(3x-1)}$

$= \lim_{x \to -2} \frac{x - 3}{3x - 1} = \frac{-2 - 3}{3(-2) - 1} = \frac{-5}{-7} = \frac{5}{7}$

$ax^2 + bx + c$

$a = 1$

$a \neq 1 \quad \times 3 \quad (6)$

$3x^2 + 5x - (2) \quad 6 \cdot 1$

$[3x^2 + 6x][-x - 2]$

$3x(x + 2) - (x + 2)$

$(x + 2)(3x - 1)$

(11) $\lim_{x \to -5} \frac{2x^2 + 9x - 5}{x^2 - 25}$ 10, -10

$f(-5) = \frac{2(-5)^2 + 9(-5) - 5}{(-5)^2 - 25} = \frac{0}{0}$

$\lim_{x \to -5} \frac{[2x^2 + 10x][-x - 5]}{x^2 - 25} = \frac{2x(x+5) - (x+5)}{(x-5)(x+5)} =$

$\frac{(x+5)(2x-1)}{(x+5)(x-5)} = \lim_{x \to -5} \frac{(2x-1)}{x-5} = \frac{2(-5) - 1}{-5 - 5} = \frac{-11}{-10} = \frac{11}{10}$

(12) $\lim_{x \to 3} \frac{t^3 - 27}{t^2 - 9}$

$f(3) = \frac{3^3 - 27}{3^2 - 9} = \frac{0}{0}$

$\lim_{x \to 3} \frac{(t-3)(t^2 + 3t + 9)}{(t-3)(t+3)} = \lim_{x \to 3} \frac{t^2 + 3t + 9}{t + 3} = \frac{9 + 9 + 9}{9}$

$= \frac{27}{9} = 3$

(13) $\lim_{u \to -1} \frac{u+1}{u^3+1} =$

$f(-1) = \frac{0}{0}$

$\lim_{x \to -1} \frac{\cancel{u+1}}{\cancel{(u+1)}(u^2-u+1)} = \lim_{x \to -1} \frac{1}{u^2-u+1} = \frac{1}{1+1+1} = \boxed{\frac{1}{3}}$

(14) $\lim_{h \to 0} \frac{\overbrace{(h-3)}^{x}{}^2 - 9}{h}$ $\qquad x^2 - 9 = (x-3)(x+3)$

$f(0) = \frac{0}{0}$

$\lim_{h \to 0} \frac{[h-3-3][h-3+3]}{h} = \frac{[h-6]\cancel{h}}{\cancel{h}}$

$\lim_{h \to 0} 0 - 6 = \boxed{-6}$

(15) $\lim_{x \to 9} \frac{9 - x}{3 - \sqrt{x}}$ $\qquad x = y^2 \quad \sqrt{x} = \quad 9 - y^2 = (3-y)(3+y)$

$f(9) = \frac{0}{0}$

$\lim_{x \to 9} \frac{\cancel{(3-\sqrt{x})}(3+\sqrt{x})}{\cancel{3-\sqrt{x}}} = \lim_{x \to 9} 3 + \sqrt{9} = \boxed{6}$

(16) $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^4 - 3x^2 - 4}$ $\qquad x^4 = (x^2)^2$

$f(2) = \frac{4 - 8 + 4}{16 - 12 - 4} = \frac{0}{0}$

$\lim_{x \to 2} \frac{(x-2)(x-2)}{(x^2-4)(x^2+1)} = \frac{(x-2)\cancel{^2}}{\cancel{(x-2)}(x+2)(x^2+1)} =$

$\lim_{x \to 2} \frac{(x-2)}{(x+2)(x^2+1)} = \frac{0}{4(5)} = \frac{0}{20} = \boxed{0}$

(17) $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

$f(16) = \dfrac{0}{0}$

$\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{x(16 - x)} = \dfrac{4 - \sqrt{x}}{x(4 - \sqrt{x})(4 + \sqrt{x})}$

$\lim_{x \to 16} \dfrac{1}{16(4 + \sqrt{16})} = \dfrac{1}{16(8)} = \boxed{\dfrac{1}{128}}$